Building Fact Fluency and Automaticity: Focus on Multiplication

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Developing proficiency with operations in whole and rational numbers enables students to spend more energy learning to reason complex mathematics. However, many students struggle to learn basic facts. The purpose of this session is to introduce specific research-based strategies for developing conceptual understanding and computational fluency of whole and rational numbers.

What is Fluency versus Automaticity?
• NRC (2001): “Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.”
• Automaticity is fast recall of a facts which seemingly appear instant.
• How much time should be devoted daily?

Why Develop Fluency and Automaticity
• Builds confidence and a predictor of future success (UVA).
• In order to solve many complex mathematics problems students must perform basic computations accurately and fluently. Those who expend too much of their cognitive capacity performing basic operations may have insufficient capacity to apply toward complex mathematics (Parkhurst et al., 2010; Woodward, 2006).
• KSDE: “Our students need a level of automaticity and fluency that allows them to explore the conceptual understandings of the ideas they encounter.”

Rigor is a deep, authentic command of mathematical concepts
• Educators will need to pursue, with equal intensity, 3 aspects of rigor in the major work of each grade:
  – Conceptual understanding
  – Procedural skill and fluency
  – Application

Experts Suggest (adapted from Riccomini, 2013)
1. Build understanding first
2. Accuracy and reasoning
3. Fluency strategies
4. Automaticity

...with a twist
Conceptual and Strategic Focus
• Counting Strategies
• Manipulative Aided counting
• Composition and Decomposition
• Pictorial Models
• Verbalization of processes
• Concrete to Representational to Abstract Sequence of Instruction

CRA Example
Use place value to add within 100
\[ 26 + 18 \]

CRA Example
Use place value to subtract within 100
\[ 33 - 18 \]

Scope and Sequence of Multiplication
Kindergarten – Numbers and early strategic counting (Number Sense)
1st grade – Counting by 10s, 2s, 5s, (multiples)
2nd grade – Completing the multiples; Introduction to “times” and groups; Missing factors
3rd grade – Single digit multiplication to automaticity; one-digit x two-digit strategies
4th grade – multi-digit multiplication to 2x3 strategies
5th grade – multi-digit multiplication to 3x3 strategies
6th grade – Rational number strategies

Establishing Fluency
Assess Accuracy before Speed &
Expect Fluency before Automaticity

Learn how to drive on a slow local road before hitting the turnpike

Building Fact Understanding
• Multiplication types (Underhill, 1981)
  – Arrays
  – Sets
  – Linear Models
  – Cross Products
  – Repeated Addition
• Fifth grade “Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.”
• \(7.6 \times 2.4 = ?\)

\[
\begin{array}{ccc}
3 & 14 & 12 \\
4 & 28 & 24 \\
\hline
14 + 1.2 + 2.8 + 0.24 &= 18.24
\end{array}
\]

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\[
\text{Areas / Arrays}
\]

\[
\text{Sets}
\]

\[
4 \times 5
\]

\[
\text{**Five**} \quad \text{**Ten**} \quad \text{**Fifteen**} \quad \text{**Twenty**}
\]

\[
= 20 \text{ total}
\]

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\[
\text{Areas / Arrays}
\]

\[
\text{Linear Models}
\]

• Incremental or Open Number Lines
Ex. 4x12

\[
\begin{array}{c}
\text{Incremental or Open Number Lines}
\end{array}
\]

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\[
\text{Fluency within an MTSS framework}
\]

• Tier 1
  – Instructional approaches
  – Schoolwide preparation and practice
  – Accommodations
• Tier 2
  – Small group preparation, practice, and remediation
• Tier 3
  – Individualized practice and remediation

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\[
\text{Considerations for fluency to automaticity}
\]

• Fluency: Understandings and relationships between facts and properties
• Automaticity: Oral/Auditory versus Visual-Response rehearsal and recall

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Emphasize Tier 1 first

- Should time be devoted to establishing fluency and automaticity in Tier 1?
- What are some guidelines to spur the development of fact fluency?
  - Time?
  - Frequency?
  - Methods?

Weekly routine for interventions

- Set up at least 10 minutes of class time when fluency will be the focus, 4-5x per week
- If the need applies to a small group, set up a space for the practice to occur
- Vary the output throughout the week
- Vary the type of fluency as the student develops proficiency

Monday
- Multiplication - isolated and silent
Tuesday
- Multiplication - conceptual and verbal with partner
Wednesday
- Multiplication - isolated and silent
Thursday
- Multiplication - isolated and oral
Friday
- Multiplication - isolated and oral game

What’s next?

Pair-share

- If students at your elementary school consistently score low in basic facts, how might you change the emphasis on learning facts?
- Example- 4th and 5th grade teachers argue that students don’t yet know their multiplication facts. However, the 3rd grade team says that they devote considerable time to fact fluency. What could you do?

Preparations – repair as you go

- If grade level standard applies to your grade, make fluency strategies part of a routine until mastered
- If not on grade level, then
  - Investigate why errors develop
  - If facts or fluency are an issue, then pinpoint the lowest level first
  - Determine if the lowest level can be combined with other areas (e.g., negatives with multiplication)
  - Focus instruction and practice on that area
  - Individualize your preparations as much as possible

Tiers 2+3 Considerations

Pair-share

- If a student is struggling in a middle school concept that is affected by fact knowledge, do you focus on the facts or the current concept?
- Example- a student struggles with solving single variable equations. Scores reveal a weakness in computation and the teacher says that the student struggles with computation.

Daily routine for fact interventions

- Dedicate at least 10 minutes daily
- Fluency is more than practice, it is instructional
- Students may not know the reasoning behind computation

- **10 n – 3 = 2n, simplify the expression**

- 2 minutes – set-up the class
- 5 minutes – conceptual and strategy practice
- 3 minutes – practice
Fluency goes beyond speed

Three elements to fluency
a) Speed of recall
b) Conceptual understanding and verbal reasoning
c) Embedded

Two types of fluency: digit computation and embedded fluency
- Digit computation: typically practiced in the form of basic problems where students do not need to analyze anything but the computational symbol (+, -, x, ÷)
- Embedded fluency: the ease with which computation can be performed within a complex math sentence.
  - Ex. In the math sentence 6x=42, a student must divide each side by 6 to determine that 1x=7.
  - In order to help students improve their embedded fluency, it is important to practice computation within forms of mathematics sentences, such as fractions, integers, distributive properties, and problems requiring understanding of the order of operations.

Embedded Fluency
Simplify the expression \( \frac{2(0y^2 + 5x - 6)}{8x^2 + 6y^3} \)

6x (3y − 6x) − 5 (7y + 1y) = 4 (8 − x), solve for y

How is fluency practice usually completed?
Pros and Cons for each
- Flash cards
- Games
- Worksheet
- Computer format (e.g., Mad Minutes)
- Daily Five
- Take home packs
- None – give a calculator

Scaffolded support (adapted from Gregorio, 2010)
- Organize facts by level of difficulty with students
  - x 1
  - x 10
  - x 2
  - x 4
  - x 5
  - x 3
  - x 9
  - x 6
  - x 7
  - x 8
Incremental Rehearsal: Don’t shred your flash cards just yet

Ratio
- Flash cards can be effective at establishing fluency and automaticity when using incremental rehearsal (Burns, 2005; Burns et al., 2014)
- Students are presented known to unknown material in a ratio of 9:1 (90% to 10%)
- In a ten card stack, this means that 9 of the answers are known and only one is yet to be learned.

Presenting unknown problems
- Students must build momentum and motivation by answering several correct in a row before an unknown problem is presented.
- When the unknown problem is presented, the answer is immediately provided.

Incremental Rehearsal (IR) Steps
1. Print flash cards for visual and auditory practice
2. Assess the student to determine which facts are known (K) and which are still unknown (U).
3. Present one unknown fact (U)
4. Practice known to unknown material at a ratio of 9:1. In a ten-card stack, this means that 9 of the answers are known and only one is yet to be learned.
5. Immediately provide the answer when the unknown problem is presented.
6. Build momentum and motivation by having the student answer several questions correctly in a row before an unknown problem is presented.

Hold IR sessions over several days, so that not too many answers are learned per day (three to five). Long-term recall is benefitted by short intervention sessions rather than long ones.

Hybrid IR
a) Introduce new fact and practice it 5 times both with and without visual cue
b) Then verbally practice it at least four times embedded in ten problems
   \[8 \times 6; 5 \times 3; 8 \times 6; 4 \times 6; 1 \times 7; 8 \times 6; 3 \times 4; 2 \times 2; 8 \times 6; 7 \times 0\]
c) Finally, practice it to written speed at least four times within a 16
   \[8 \times 6; 8 \times 6; 3 \times 4; 2 \times 2; 8 \times 6; 7 \times 0\]

Drill sandwich (Browder & Roberts, 1993)
1) Make or obtain a set of flashcards for the information to be learned.
2) Go through the complete stack of cards once, separating the list into two piles – knowns and unknowns.
3) Build a “sandwich” using seven knowns (K) and three unknowns (U).
   - Be sure to follow the pattern (K-K-K-U-K-U-K-U)
4) Have the child/children practice identifying all ten items in the order above.
5) As the unknown items are learned so that the response is immediate and automatic, move them into a known section of the sandwich by removing repeated accurate knowns.
6) Add new unknowns and repeat.

This approach adds 3 new facts daily for a total of 12 by a summative assessment on Friday.

Cover-Copy-Compare (Skinner, McLaughlin, & Logan, 1997)
https://www.youtube.com/watch?v=WjlkAi44qTY

Preparation:
- The teacher selects up to 10 math facts for the student to practice during the session and writes those facts (including number sentence and answer) as correct models into the left column of the Cover-Copy-Compare Worksheet.
- The teacher then pre-folds the sheet using as a guide the vertical dashed line (‘fold line’) dividing the left side of the student worksheet.

6
Cover-Copy-Compare Steps

1) Study the correctly completed math fact (model) that appears in the left column of the sheet.
2) Fold the left side of the page over at the pre-folded vertical crease to hide the original math fact ('Cover').
3) Copy from memory the math fact and answer, writing it in the first response blank under the 'Student Response' section of the Cover-Copy-Compare worksheet ('Copy').

Teacher uncovers the original correct model and compares it to the student response ('Compare').

IF CORRECT, the student moves to the next item on the list and repeats these procedures.

IF INCORRECT, the student draws a line through the incorrect response, and repeats earlier steps and again checks the correctness of the copied item.
Continue until all math facts on the sheet have been copied and checked against the correct models.

Detect, Practice, Repair (Parkhurst et al., 2010)

Detect, Practice, Repair (DPR) is a multi-component, class-wide procedure that focuses on enhancing fluency by allowing students to practice those math facts that they have not developed to the point of automaticity (Poncy et al. 2006). Parkhurst et al (2010) improved multiplication facts with 5th grade students in 6-10 trials.

Detect = During a detect phase, Poncy et al. used a metronome to pace a group of students through a series of math facts, with the metronome signaling 1.5 s intervals to respond to each fact.
Practice = After this paced assessment, each student circled those problems that he/she did not answer and then applied the practice phase to those identified problems by performing the Cover, Copy, Compare (CCC) procedure.
Repair = Corrective feedback and repeat practice

Schema-based approaches

Some studies found that word problem approaches led to increased performance on fluency measures (Hembd et al., 2007)

What do we do in the meantime for students who haven’t mastered their facts?

• Calculator?
  – “The Panel cautions that to the degree that calculators impede the development of automaticity, fluency in computation will be adversely affected” (NMAP, 2008, p. xxiv)

• Options?

Hundreds Table Accommodation

• Mix the accommodation with fluency intervention
• Slowly fade the utility of the table by covering what has been “mastered”
• Make the table more cumbersome to use as the student progresses

Hundreds Table Accommodation

A Modification
Adaptations

- How could I help a student practice fluency/automaticity if they are in MS or HS?
- How could these approaches be adapted to fractions or integer computation?
Summation

- What is a difference between fluency and automaticity?
- What should be developed along with fluency?
- How many new facts should be mastered at a time?
- When you teach a new fact what else should be done?
- How many times a week should you practice fluency or automaticity?
- How much time a day should be allotted to fluency/automaticity?

Sample Resources


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