Teaching Math to Young Children

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The goal of this practice guide is to offer educators specific, evidence-based recommendations that address the challenge of teaching early math to children ages 3 to 6. The guide provides practical, clear information on critical topics related to teaching early math and is based on the best available evidence as judged by the authors.

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IES Practice Guide

Teaching Math to Young Children

November 2013

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What Works Clearinghouse practice guide citations begin with the panel chair, followed by the names of the panelists listed in alphabetical order.

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Overview of Recommendations

Recommendation 1.

Teach number and operations using a developmental progression.

- First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.
- Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.
- Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.
- Encourage children to label collections with number words and numerals.
- Once children develop these fundamental number skills, encourage them to solve basic problems.

Recommendation 2.

Teach geometry, patterns, measurement, and data analysis using a developmental progression.

- Help children to recognize, name, and compare shapes, and then teach them to combine and separate shapes.
- Encourage children to look for and identify patterns, and then teach them to extend, correct, and create patterns.
- Promote children's understanding of measurement by teaching them to make direct comparisons and to use both informal or nonstandard (e.g., the child's hand or foot) and formal or standard (e.g., a ruler) units and tools.
- Help children to collect and organize information, and then teach them to represent that information graphically.

Recommendation 3.

Use progress monitoring to ensure that math instruction builds on what each child knows.

- Use introductory activities, observations, and assessments to determine each child's existing math knowledge, or the level of understanding or skill he or she has reached on a developmental progression.
- Tailor instruction to each child's needs, and relate new ideas to his or her existing knowledge.
- Assess, record, and monitor each child's progress so that instructional goals and methods can be adjusted as needed.

Recommendation 4.

Teach children to view and describe their world mathematically.

- Encourage children to use informal methods to represent math concepts, processes, and solutions.
- Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences.
- Use open-ended questions to prompt children to apply their math knowledge.
- Encourage children to recognize and talk about math in everyday situations.

Recommendation 5.

Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

- Plan daily instruction targeting specific math concepts and skills.
- Embed math in classroom routines and activities.
- Highlight math within topics of study across the curriculum.
- Create a math-rich environment where children can recognize and meaningfully apply math.
- Use games to teach math concepts and skills and to give children practice in applying them.

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Institute of Education Sciences Levels of Evidence for Practice Guides

This section provides information about the role of evidence in Institute of Education Sciences' (IES) What Works Clearinghouse (WWC) practice guides. It describes how practice guide panels determine the level of evidence for each recommendation and explains the criteria for each of the three levels of evidence (strong evidence, moderate evidence, and minimal evidence).

The level of evidence assigned to each recommendation in this practice guide represents the panel's judgment of the quality of the existing research to support a claim that, when these practices were implemented in past research, favorable effects were observed on student outcomes. After careful review of the studies supporting each recommendation, panelists determine the level of evidence for each recommendation using the criteria in Table 1. The panel first considers the relevance of individual studies to the recommendation and then discusses the entire evidence base, taking the following into consideration:

- the number of studies
- the study designs
- the internal validity of the studies
- whether the studies represent the range of participants and settings on which the recommendation is focused
- whether findings from the studies can be attributed to the recommended practice
- whether findings in the studies are consistently positive

A rating of *strong evidence* refers to consistent evidence that the recommended strategies, programs, or practices improve student outcomes for a diverse population of students.[1] In other words, there is strong causal and generalizable evidence.

A rating of *moderate evidence* refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated) or to evidence from studies that are generalizable but have some causal ambiguity. It also might be that the studies that exist do not specifically examine the outcomes of interest in the practice guide, although they may be related.

A rating of *minimal evidence* suggests that the panel cannot point to a body of research that demonstrates the practice's positive effect on student achievement. In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion;[2] in other cases, it means that researchers have not yet studied this practice, or that there is weak or conflicting evidence of effectiveness. A minimal evidence rating does not indicate that the recommendation is any less important than other recommendations with a strong or moderate evidence rating.

In developing the levels of evidence, the panel considers each of the criteria in Table 1. The level of evidence rating is determined by the lowest rating achieved for any individual criterion. Thus, for a recommendation to get a strong rating, the research must be rated as strong on each criterion. If at least one criterion receives a rating of moderate and none receive a rating of minimal, then the level of evidence is determined to be moderate. If one or more criteria receive a rating of minimal, then the level of evidence is determined to be minimal.

Table 1. Institute of Education Sciences levels of evidence for practice guides

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Validity	High internal validity (high-quality causal designs). Studies must meet WWC standards with or without reservations.[3] **AND** High external validity (requires multiple studies with high-quality causal designs that represent the population on which the recommendation is focused). Studies must meet WWC standards with or without reservations.	High internal validity but moderate external validity (i.e., studies that support strong causal conclusions but generalization is uncertain). **OR** High external validity but moderate internal validity (i.e., studies that support the generality of a relation but the causality is uncertain).[4]	The research may include evidence from studies that do not meet the criteria for moderate or strong evidence (e.g., case studies, qualitative research).
Effects on relevant outcomes	Consistent positive effects without contradictory evidence (i.e., no statistically significant negative effects) in studies with high internal validity.	A preponderance of evidence of positive effects. Contradictory evidence (i.e., statistically significant negative effects) must be discussed by the panel and considered with regard to relevance to the scope of the guide and intensity of the recommendation as a component of the intervention evaluated.	There may be weak or contradictory evidence of effects.
Relevance to scope	Direct relevance to scope (i.e., ecological validity)—relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated.	Relevance to scope (ecological validity) may vary, including relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated. At least some research is directly relevant to scope (but the research that is relevant to scope does not qualify as strong with respect to validity).	The research may be out of the scope of the practice guide.
Relationship between research and recommendations	Direct test of the recommendation in the studies or the recommendation is a major component of the intervention tested in the studies.	Intensity of the recommendation as a component of the interventions evaluated in the studies may vary.	Studies for which the intensity of the recommendation as a component of the interventions evaluated in the studies is low; and/or the recommendation reflects expert opinion based on reasonable extrapolations from research.

(continued)

Table 1. Institute of Education Sciences levels of evidence for practice guides *(continued)*

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Panel confidence	Panel has a high degree of confidence that this practice is effective.	The panel determines that the research does not rise to the level of strong but is more compelling than a minimal level of evidence. Panel may not be confident about whether the research has effectively controlled for other explanations or whether the practice would be effective in most or all contexts.	In the panel's opinion, the recommendation must be addressed as part of the practice guide; however, the panel cannot point to a body of research that rises to the level of moderate or strong.
Role of expert opinion	Not applicable	Not applicable	Expert opinion based on defensible interpretations of theory (theories). (In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion; in other cases, it means that researchers have not yet studied this practice.)
When assessment is the focus of the recommendation	For assessments, meets the standards of *The Standards for Educational and Psychological Testing*.[5]	For assessments, evidence of reliability that meets *The Standards for Educational and Psychological Testing* but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.	Not applicable

The panel relied on WWC evidence standards to assess the quality of evidence supporting educational programs and practices. The WWC evaluates evidence for the causal validity of instructional programs and practices according to WWC standards. Information about these standards is available at http://whatworks.ed.gov. Eligible studies that meet WWC evidence standards for group designs or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages.

Introduction to the *Teaching Math to Young Children* Practice Guide

Children demonstrate an interest in math well before they enter school.[6] They notice basic geometric shapes, construct and extend simple patterns, and learn to count. The *Teaching Math to Young Children* practice guide presents five recommendations designed to capitalize on children's natural interest in math to make their preschool and school experience more engaging and beneficial. These recommendations are based on the panel members' expertise and experience and on a systematic review of the available literature. The first two recommendations identify which early math content areas[7] (number and operations, geometry, patterns, measurement, and data analysis)[8] should be a part of the preschool, prekindergarten, and kindergarten curricula, while the last three recommendations discuss strategies for incorporating this math content in classrooms. The recommendations in this guide can be implemented using a range of resources, including existing curricula.

In recent years, there has been an increased emphasis on developing and testing new early math curricula.[9] The development of these curricula was informed by research focused on the mechanisms of learning math,[10] and recent studies that test the impact of early math curricula show that devoting time to specific math activities as part of the school curriculum is effective in improving children's math learning before and at the beginning of elementary school.[11] Research evidence also suggests that children's math achievement when they enter kindergarten can predict later reading achievement; foundational skills in number and operations may set the stage for reading skills.[12]

Despite these recent efforts, many children in the United States lack the opportunity to develop the math skills they will need for future success. Research indicates that individual differences among children are evident before they reach school.[13] Children who begin with relatively low levels of math knowledge tend to progress more slowly in math and fall further behind.[14] In addition to these differences within the United States, differences in achievement between American children and students in other countries can be observed as early as the start of kindergarten.[15] Low achievement at such an early age puts U.S. children at a disadvantage for excelling in math in later years.[16] The panel believes that the math achievement of young children can be improved by placing more emphasis on math instruction throughout the school day.

This practice guide provides concrete suggestions for how to increase the emphasis on math instruction. It identifies the early math content areas that are important for young children's math development and suggests instructional techniques that can be used to teach them.

The panel's recommendations are in alignment with state and national efforts to identify what children should know, such as the Common Core State Standards (CCSS) and the joint position statement from the National Association for the Education of Young Children (NAEYC) and National Council of Teachers of Math (NCTM).[17] The early math content areas described in Recommendations 1 and 2 align with the content area objectives for kindergartners in the CCSS.[18] The panel recommends teaching these early math content areas using a developmental progression, which is consistent with the NAEYC/NCTM's recommendation to use curriculum based on known sequencing of mathematical ideas. Some states, such as New York, have adopted the CCSS and developed preschool standards that support the CCSS. *The New York State Foundation to the Common Core* is guided by principles that are similar to recommendations in this guide.[19]

The recommendations also align with the body of evidence in that the recommended practices are frequently components of curricula that are used in preschool, prekindergarten, and kindergarten classrooms. However, the practices are part of a larger curriculum, so their

effectiveness has not been examined individually. As a result, the body of evidence does not indicate whether each recommendation would be effective if implemented alone. However, the evidence demonstrates that when all of the recommendations are implemented together, students' math achievement improves.[20] Therefore, the panel suggests implementing all five recommendations in this guide together to support young children as they learn math. The first two recommendations identify important content areas. Recommendation 1 identifies number and operations as the primary early math content area, and Recommendation 2 describes the importance of teaching four other early math content areas: geometry, patterns, measurement, and data analysis. Recommendations 3 and 4 outline how teachers can build on young children's existing math knowledge, monitor progress to individualize instruction, and eventually connect children's everyday informal math knowledge to the formal symbols that will be used in later math instruction. Finally, Recommendation 5 provides suggestions for how teachers can dedicate time to math each day and link math to classroom activities throughout the day.

Scope of the practice guide

Audience and grade level. This guide is intended for the many individuals involved in the education of children ages 3 through 6 attending preschool, prekindergarten, and kindergarten programs. Teachers of young children may find the guide helpful in thinking about what and how to teach to prepare children for later math success. Administrators of preschool, prekindergarten, and kindergarten programs also may find this guide helpful as they prepare teachers to incorporate these early math content areas into their instruction and use the recommended practices in their classrooms. Curriculum developers may find the guide useful when developing interventions, and researchers may find opportunities to extend or explore variations in the body of evidence.

Common themes. This guide highlights three common themes for teaching math to young children.

- **Early math instruction should include multiple content areas.** Understanding the concept of number and operations helps create the foundation of young children's math understanding, and is the basis for Recommendation 1. Because there is much more to early math than understanding number and operations, the panel also reviewed the literature on instruction in geometry, patterns, measurement, and data analysis, as summarized in Recommendation 2. Giving young children experience in early math content areas other than number and operations helps prepare them for the different math subjects they will eventually encounter in school, such as algebra and statistics, and helps them view and understand their world mathematically.

- **Developmental progressions can help guide instruction and assessment.** The order in which skills and concepts build on one another as children develop knowledge is called a developmental progression. Both Recommendation 1 and Recommendation 2 outline how various early math content areas should be taught according to a developmental progression. There are different developmental progressions for each skill. These developmental progressions are important for educators to understand because they show the order in which young children typically learn math concepts and skills. The panel believes educators should pay attention to the order in which math instruction occurs and ensure that children are comfortable with earlier steps in the progression before being introduced to more complex steps. Understanding developmental progressions is also necessary to employ progress monitoring, a form of assessment that tracks individual children's success along the steps in the progression, as described in Recommendation 3.[21] The panel developed a specific developmental progression for

teaching number and operations based on their expertise and understanding of the research on how children learn math (see Table 3). The panel acknowledges that different developmental progressions exist; for example, the *Building Blocks* curriculum is based on learning trajectories that are similar but not identical to the developmental progression presented.[22] For a discussion of learning trajectories in mathematics broadly, as well as the connection between learning trajectories, instruction, assessment, and standards, see Daro, Mosher, and Corcoran (2011).

Developmental progressions refer to sequences of skills and concepts that children acquire as they build math knowledge.

- **Children should have regular and meaningful opportunities to learn and use math.** The panel believes that math should be a topic of discussion throughout the school day and across the curriculum. Early math instruction should build on children's current understanding and lay the foundation for the formal systems of math that will be taught later in school. These instructional methods guide Recommendations 4 and 5, which focus on embedding math instruction throughout the school day.[23]

Summary of the recommendations

Recommendation 1 establishes number and operations as a foundational content area for children's math learning. The recommendation presents strategies for teaching number and operations through a developmental progression. Teachers should provide opportunities for children to subitize small collections, practice counting, compare the magnitude of collections, and use numerals to quantify collections. Then, teachers should encourage children to solve simple arithmetic problems.

Recommendation 2 underscores the importance of teaching other early math content areas—specifically geometry, patterns, measurement, and data analysis—in preschool, prekindergarten, and kindergarten. The panel reiterates the importance of following a developmental progression to organize the presentation of material in each early math content area.

Recommendation 3 describes the use of progress monitoring to tailor instruction and build on what children know. The panel recommends that instruction include first determining children's current level of math knowledge based on a developmental progression and then using the information about children's skills to customize instruction. Monitoring children's progress throughout the year can then be an ongoing part of math instruction.

Recommendation 4 focuses on teaching children to view their world mathematically. The panel believes children should begin by using informal methods to represent math concepts and then learn to link those concepts to formal math vocabulary and symbols (such as the word *plus* and its symbol, +). Teachers can use open-ended questions and math conversation as a way of helping children to recognize math in everyday situations.

Recommendation 5 encourages teachers to set aside time each day for math instruction and to look for opportunities to incorporate math throughout the school day and across the curriculum.

Summary of supporting research

The panel used a substantial amount of national and international[24] research to develop this practice guide. This research was used to inform the panel's recommendations and to rate the level of evidence for the effectiveness of these recommendations. In examining the research base for practices and strategies for teaching math to young children, the panel paid particular attention to experimental and quasi-experimental studies that meet What Works Clearinghouse (WWC) standards.

The panel considered two bodies of literature to develop the recommendations in the practice guide: (1) theory-driven research, including developmental research[25] and (2) research on effective practice. The theory-driven research provided a foundation from which the panel developed recommendations by providing an understanding of how young children learn math. As this first body of literature did not examine the effectiveness of interventions, it was not reviewed under WWC standards, but it did inform the panel's expert opinion on how young children learn math. The second body of literature provided evidence of the effectiveness of practices as incorporated in existing interventions. This body of literature was eligible for review under WWC standards and, along with the panel's expert opinion, forms the basis for the levels of evidence assigned to the recommendations.

Recommendations were developed in an iterative process. The panel drafted initial recommendations that were based on its expert knowledge of the research on how young children learn math. The WWC then conducted a systematic review of literature following the protocol to identify and review the effectiveness literature relevant to teaching math to young children. The findings of the systematic review were then evaluated to determine whether the literature supported the initial recommendations or suggested other practices that could be incorporated in the recommendations. The final recommendations, which are presented in this guide, reflect the panel's expert opinion and interpretation of both bodies of literature.

The research base for this guide was identified through a comprehensive search for studies evaluating instructional practices for teaching math to children in preschool, prekindergarten, or kindergarten programs. The *Scope of the practice guide* section (p. 8) describes some of the criteria and themes used as parameters to help shape the literature search. A search for literature related to early math learning published between 1989 and 2011 yielded more than 2,300 citations. Of the initial set of studies, 79 studies used experimental and quasi-experimental designs to examine the effectiveness of the panel's recommendations. From this subset, 29 studies met WWC standards and were related to the panel's recommendations.[26]

The strength of the evidence for the five recommendations varies, and the level of evidence ratings are based on a combination of a review of the body of evidence and the panel's expertise. The supporting research provides a moderate level of evidence for Recommendation 1 and a minimal level of evidence for Recommendations 2–5. Although four recommendations were assigned a minimal level of evidence rating, all four are supported by studies with positive effects. These studies include a combination of practices that are covered in multiple recommendations; therefore, it was not possible to attribute the effectiveness of the practice to any individual recommendation.[27] For example, teaching the content area of number and operations, along with other math content areas like geometry, patterns, and data analysis, was often a common component of effective comprehensive curricula. Additionally, while the panel suggests that teachers assess children's understanding on a regular basis and use that information to tailor instruction, the panel could not find research that isolated the impact of progress monitoring on children's math knowledge. Similarly, there is limited evidence on the effectiveness of teaching children to view and describe their world mathematically, as this component was never separated from other aspects of the intervention. Finally, there also is limited evidence on the effectiveness of time spent on math because there is a lack of research in which the only difference between groups was instructional time for math.

Although the research base does not provide direct evidence for all recommendations in isolation, the panel believes the recommendations in this guide are necessary components of early math instruction based on panel

members' knowledge of and experience working in preschool, prekindergarten, and kindergarten classrooms. The panel identified evidence indicating that student performance improves when these recommendations are implemented together.

Table 2 shows each recommendation and the level of evidence rating for each one as determined by the panel. Following the recommendations and suggestions for carrying out the recommendations, Appendix D presents more information on the body of evidence supporting each recommendation.

Table 2. Recommendations and corresponding levels of evidence

	Levels of Evidence		
Recommendation	**Strong Evidence**	**Moderate Evidence**	**Minimal Evidence**
1. Teach number and operations using a developmental progression.		◆	
2. Teach geometry, patterns, measurement, and data analysis using a developmental progression.			◆
3. Use progress monitoring to ensure that math instruction builds on what each child knows.			◆
4. Teach children to view and describe their world mathematically.			◆
5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day.			◆

Teach number and operations using a developmental progression.

Early experience with number and operations is fundamental for acquiring more complex math concepts and skills.[28] In this recommendation, the panel describes the main aspects of early number knowledge, moving from basic number skills to operations.

Effective instruction depends on identifying the knowledge children already possess and building on that knowledge to help them take the next developmental step. Developmental progressions can help identify the next step by providing teachers with a road map for developmentally appropriate instruction for learning different skills.[29] For example, teachers can use progressions to determine the developmental prerequisites for a particular skill and, if a child achieves the skill, to help determine what to teach next. Similarly, when a child is unable to a grasp a concept, developmental prerequisites can inform a teacher what skills a child needs to work on to move forward. In other words, developmental progressions can be helpful aids when tailoring instruction to individual needs, particularly when used in a deliberate progress monitoring process (see Recommendation 3). Although there are multiple developmental progressions that may vary in their focus and exact ordering,[30] the steps in this recommendation follow a sequence that the panel believes represents core areas of number knowledge (see Table 3).[31] Additional examples of developmental progressions may be found in early math curricula, assessments, and research articles.

With each step in a developmental progression, children should first focus on working with small collections of objects (one to three items) and then move to progressively larger collections of objects. Children may start a new step with small numbers before moving to larger numbers with the previous step.[32]

Table 3. Examples of a specific developmental progression for number knowledge

Developmental Progression

Subitizing (small-number recognition)	Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as non-examples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three.
	A child is ready for the next step when, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful object counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle).
	A child is ready for the next step when, for example, if given five blocks and asked, "How many?" he or she counts by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five."
Counting-based comparisons of collections larger than three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., "seven" items is more than "six" items because you have to count further).
	A child is ready for the next step when he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more).
Number-after knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one.
	A child is ready for the next step when he or she can answer questions such as, "What comes after five?" by stating "five, six" or simply "six" instead of, say, counting "one, two, … six."
Mental comparisons of close or neighboring numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that "nine" is larger than "eight").
	A child has this knowledge when he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers.
Number-after equals one more	Once children can mentally compare numbers and see that "two" is one more than "one" and that "three" is one more than "two," they can conclude that any number in the counting sequence is exactly one more than the previous number.
	A child is ready for the next step when he or she recognizes, for example, that "eight" is one more than "seven."

Summary of evidence: Moderate Evidence

The panel assigned a rating of *moderate evidence* to this recommendation based on their expertise and 21 randomized controlled trials[33] and 2 quasi-experimental studies[34] that met WWC standards and examined interventions that included targeted instruction in number and operations. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The research shows a strong pattern of positive effects on children's early math achievement across a range of curricula with a focus on number and operations. Eleven studies evaluated the effectiveness of instruction in only number and operations, and all 11 studies found at least one positive effect on basic number concepts or operations.[35] The other 12 studies evaluated the effectiveness of instruction in number and operations in the context of broader curricula.

None of the 23 studies that contributed to the body of evidence for Recommendation 1 evaluated the effectiveness of instruction based on a developmental progression compared to instruction that was not guided by a developmental progression. As a result, the panel could not identify evidence for teaching based on any particular developmental progression. Additional research is needed to identify the developmental progression that reflects how most children learn math. Yet based on their expertise, and the pattern of positive effects for interventions guided by a developmental progression, the panel recommends the use of a developmental progression to guide instruction in number and operations.[36]

Positive effects were found even in studies in which the comparison group also received instruction in number and operations.[37] The panel classified an intervention as having a focus on number and operations if it included instruction in at least one concept related to number and operations. The panel found that the math instruction received by the comparison group differed across the studies, and in some cases the panel was unable to determine what math instruction the comparison group received.[38] Despite these limitations, the panel believes interventions with a focus on number and operations improve the math skills of young children.

Although the research tended to show positive effects, some of these effects may have been driven by factors other than the instruction that was delivered in the area of number and operations. For example, most interventions included practices associated with multiple recommendations in this guide (also known as multi-component interventions).[39] As a result, it was not possible to determine whether findings were due to a single practice—and if so, which one—or a combination of practices that could be related to multiple recommendations in this guide. While the panel cannot determine whether a single practice or combination of practices is responsible for the positive effects seen, the pattern of positive effects indicates instruction in teaching number and operations will improve children's math skills.

The panel identified five suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.

Being able to correctly determine the number of objects in a small collection is a critical skill that children must develop to help them learn more complex skills, including counting larger collections and eventually adding and subtracting. To give children experience with subitizing[40] (also known as small-number recognition), teachers should ask children to answer the question "How many (name of object) do you see?" when looking at collections of one to three objects.[41] As described in the first step of Table 3, children should practice stating the total for small collections without necessarily counting. Research indicates that young children can learn to use subitizing to successfully determine the quantity of a collection.[42]

Transitions between classroom activities can provide quick opportunities for children to practice subitizing. Teachers can find collections of two or three of the same object around the classroom (e.g., fingers, unit cubes, seashells, chips). Teachers can ask "How many ____?" (without counting) before transitioning to the next activity. Another way to help children practice immediately recognizing quantities is during snack time, when, for example, a teacher can give a child two crackers and then ask the child how many crackers he or she has. Practicing subitizing in meaningful, everyday contexts such as snack time, book reading, and other activities can reinforce children's math skills.

Children can also practice subitizing while working in small groups. The *Basic Hiding* game is one example of a subitizing activity that can be used with small groups of children (see Example 1).

Once children have some experience recognizing and labeling small collections of similar objects (e.g., three yellow cubes), teachers can introduce physically dissimilar items of the same type (e.g., a yellow cube, a green cube, and a red cube). Eventually, teachers can group unrelated items (e.g., a yellow cube, a toy frog, and a toy car) together and ask children, "How many?" Emphasizing that collections of three similar objects and three dissimilar objects are both "three" will help children construct a more abstract or general concept of number.[44]

As children begin to learn these concepts, they may overgeneralize. Early development is often marked by the overgeneralization of terms (e.g., saying "two" and then "three" or another number such as "five" to indicate "many").[45] The panel believes one way to help children define the limits of a number concept is to contrast examples of a number with non-examples. For instance, in addition to labeling three toys as "three," labeling four toys as "not three" (e.g., "That's four toys, not three toys") can help children clearly understand the meaning of "three." Once children are accustomed to hearing adults labeling examples and non-examples, teachers can have children find their own examples and non-examples (e.g., "Can someone find two toys? Now, what is something that is *not* two?").[46]

Example 1. The *Basic Hiding* game[43]

Objective

Practice subitizing—immediately recognizing and labeling small numbers and constructing a concept of one to three—and the concept of number constancy (rearranging items in a set does not change its total).

Materials needed:

- Objects. Use a small set of identical objects early on and later advance to larger sets or sets of similar, but not identical, objects.
- Box, cloth, or other item that can be used to hide the objects.

Directions: With a small group of children, present one to three objects on a mat for a few seconds. Cover them with a cloth or box and then ask the children, "Who can tell me how many (name of objects) I am hiding?" After the children have answered, uncover the objects so that the objects can be seen. The children can count to check their answer, or the teacher can reinforce the answer by saying, for example, "Yes, two. See, there are two (objects) on the mat: one, two." Continue the game with different numbers of objects arranged in different ways. Teachers can also tailor the *Basic Hiding* game for use with the whole class or individual children.

Early math content areas covered

- Subitizing
- Increasing magnitude up to five items

Monitoring children's progress and tailoring the activity appropriately

- Vary the number of objects to determine whether children are ready to use larger sets.
- If a child has difficulty, before covering the objects, ask the child how many items he or she sees. Then, cover the objects and ask again. For larger collections (greater than three), allow the child to check his or her answer by counting.

Integrating the activity into other parts of the day

- Consider playing the game at various points during the day with different sets of objects, including objects that are a part of children's everyday experience (e.g., spoons and blocks).

Using the activity to increase math talk in the classroom

- Use both informal ("more" or "less") and formal ("add" and "subtract") language to describe changing the number of objects in the set.

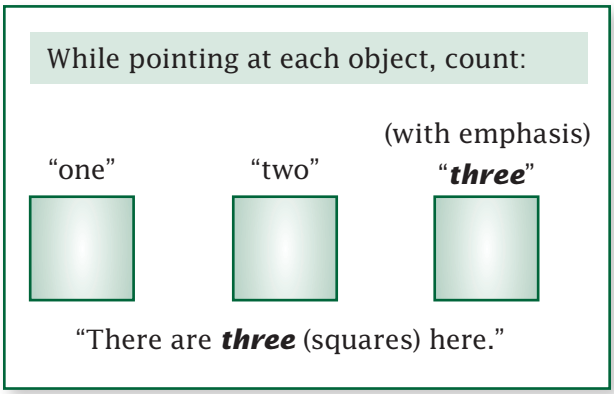
2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.

Small-number recognition provides a basis for learning the one-to-one counting principle in a meaningful manner.[47] Often, children begin learning about number from an early age by reciting the count sequence ("one, two, three, four…"). But learning to assign the numbers of the count sequence to a collection of objects that are being counted can be a challenging step. Once children are able to reliably recognize and label collections of one to three items immediately (without counting), they have started to connect numbers with quantity. As illustrated in the second step of Table 3, they should then begin to use one-to-one counting to identify "how many" are in larger collections.[48]

To count accurately, one—and only one—number word must be assigned to each item in the collection being counted. For example, when counting four pennies, children must point to a penny and say "one," point to a second penny and say "two," point to a third penny and say "three," and point to the final penny and say "four." During this activity the child will need to keep track of which pennies have been labeled and which still need to be labeled. The child can also practice recognition of the cardinality principle: that the last number word is the total (cardinal value) of the collection. Although children can learn to count one-to-one by rote, they typically do not recognize at the outset that the goal of this skill is to specify the total of a collection or how many there are. For example, when asked how many they just counted, some children count again or just guess. By learning one-to-one counting with small collections that they already recognize, children can see that the last word used in the counting process is the same as the total.[49]

Teachers should model one-to-one counting with one to three items—collections children can readily recognize and label—and emphasize or repeat the last number word used in the counting process, as portrayed in Figure 1.[50] By practicing with small collections they can already recognize, preschool, prekindergarten, and kindergarten children will begin to learn that counting is a method for answering the question, "How many?"[51]

Figure 1. Modeling one-to-one counting with one to three items



Once children can find the total with small collections, they are ready to count larger collections (four to ten objects). For example, by counting seven objects one by one ("one, two, three, four, five, six, and seven"), the child figures out that "seven" is the total number of objects in the set. Teachers can also challenge children by having them count sounds (e.g., clapping a certain number of times and asking, "How many claps?") or actions (e.g., counting the number of hops while hopping on one foot).

Children can use everyday situations and games, such as *Hidden Stars* (see Example 2), to practice counting objects and using the last number counted to determine the total quantity. This game is similar to the *Basic Hiding* game; however, in *Hidden Stars*, the goal is to count the objects first and then use that number to determine the total quantity (without recounting). It is important to demonstrate that counting is not dependent upon the order of the objects. That is, children can start from the front of a line of blocks or from the back of a line of blocks, and as long as they use one-to-one counting, they will get the same quantity.

Example 2. The *Hidden Stars* game[52]

Objective

Practice using one-to-one counting and the final number counted to identify "how many" objects.

Materials needed:

- Star stickers in varying quantities from one to ten, glued to 5-by-8-inch cards
- Paper for covering cards

Directions: Teachers can tailor the *Hidden Stars* game for use with the whole class, a small group, or individual children. Show children a collection of stars on an index card. Have one child count the stars. Once the child has counted the stars correctly, cover the stars and ask, "How many stars am I hiding?"

Early math content areas covered

- Counting
- Cardinality (using the last number counted to identify the total in the set)

Monitoring children's progress and tailoring the activity appropriately

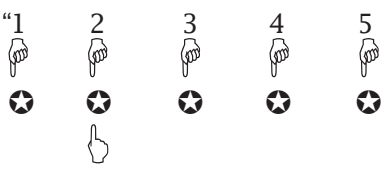
- Work with children in a small group, noting each child's ability to count the stars with accuracy and say the amount using the cardinality principle (the last number counted represents the total).
- When children repeat the full count sequence, model the cardinality principle. For example, for four items, if a child repeats the count sequence, say, "One, two, three, *four*. So I need to remember *four*. There are four stars hiding."
- Have a child hide the stars while telling him or her how many there are, emphasizing the last number as the significant number.

Using the activity to increase math talk in the classroom

- Ask, "How many?" (e.g., "How many blocks did you use to build your house? How many children completed the puzzle?")

Errors in counting. When children are still developing counting skills, they will often make errors. Some errors are predictable. For example, some children will point to the same object more than once or count twice while pointing at only one object. Table 4 describes common counting errors and provides suggestions teachers can use to correct those errors when working with children in one-on-one or small-group situations.[53]

Table 4. Common counting errors

Type of Counting Error	Example	Remedy
SEQUENCE ERROR		
Saying the number sequence out of order, skipping numbers, or using the same number more than once.	"1 2 3 6 10" 	Practice reciting (or singing) the single-digit sequence, first focusing on one to ten, then later moving on to numbers greater than ten.
Struggling with the count sequence past twelve.	Skips 15: "1…13, 14, 16, 17, 18." Uses incorrect words: "1…13, 14, fiveteen." "1…18, 19, 10-teen" or "1…29, 20-ten, 20-eleven." Stops at a certain number: "1…20" (stops) "1…20" (starts from 1 again)	Highlight and practice exceptions, such as *fif + teen*. Fifteen and thirteen are commonly skipped because they are irregular. Recognize that a nine signals the end of a series and that a new one needs to begin (e.g., nineteen marks the end of the teens). Recognize that each new series (decade) involves combining a decade and the single-digit sequence, such as twenty, twenty plus one, twenty plus two, etc. Recognize the decade term that begins each new series (e.g., twenty follows nineteen, thirty follows twenty-nine, and so forth). This involves both memorizing terms such as ten, twenty, and thirty by rote and recognizing a pattern: "add *-ty* to the single-digit sequence" (e.g., *six + ty*, *seven + ty*, *eight + ty*, *nine + ty*).
COORDINATION ERROR		
Labeling an object with more than one number word.	"1 2 3 4 5,6" 	Encourage the child to slow down and count carefully. Underscore that each item needs to be tagged only once with each number word.
Pointing to an object but not counting it.	 "1 2 3 4"	Same as above.
KEEPING TRACK ERROR		
Recounting an item counted earlier.	"1 2 3 4 5 6" 	Help the child devise strategies for sorting counted items from uncounted items. For movable objects, for instance, have the child place counted items aside in a pile clearly separated from uncounted items. For pictured objects, have him or her cross off items as counted.
SKIM		
No effort at one-to-one counting or keeping track.	Waves finger over the collection like a wand (or jabs randomly at the collection) while citing the counting sequence (e.g., "1, 2, 3…9, 10").	Underscore that each item needs to be tagged with one and only one number word and help the child to learn processes for keeping track. Model the counting.
NO CARDINALITY RULE		
Not recognizing that the last number word used in the counting process indicates the total.	Asked how many, the child tries to recount the collection or simply guesses.	Play *Hidden Stars* with small collections of one to three items first and then somewhat larger collections of items.

3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.

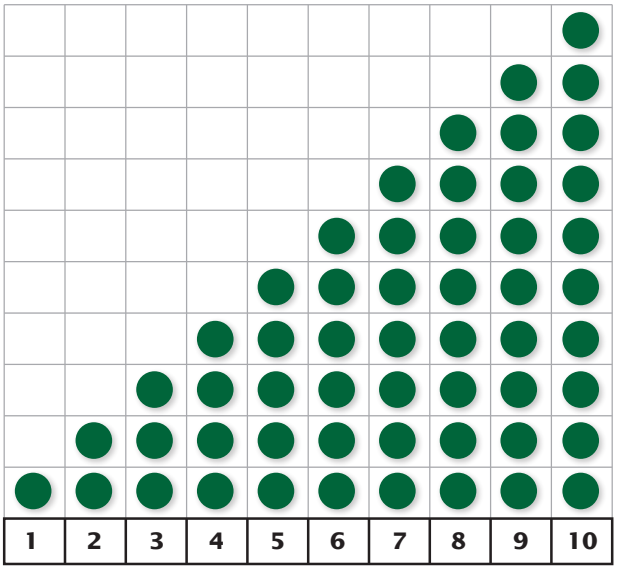
Once children can reliably determine how many objects are in a collection, either by subitizing or counting, teachers can provide them with opportunities to compare the magnitudes of different collections using number words (steps 3 through 6 in the developmental progression illustrated in Table 3).

To prepare children for making meaningful, verbal comparisons of magnitudes, teachers should ensure that they understand relational terms such as "more" and "fewer."[54] For example, a teacher can present two plates with obviously different numbers of cookies and ask, "Which plate has more cookies?" Teachers can also provide children with examples of "equal" by showing two groups with the same quantity of objects. Using these words provides children with the vocabulary for comparing larger collections.

Once children are comfortable making verbal comparisons, teachers should encourage them to use counting to compare the magnitudes of two collections.[55] Teachers can demonstrate that number words further along in the counting sequence represent larger collections.[56] Described in the third step of the developmental progression illustrated in Table 3, this is also known as the "increasing magnitude principle." A cardinality chart, as shown in Figure 2, visually underscores this principle and can be a useful tool to help children make number comparisons. Teachers can use the cardinality chart to demonstrate that the next number in the counting sequence is exactly one more than the previous number. Children can also use cardinality charts to reinforce the concepts of number-after relations, mental comparison of neighboring numbers, and the increasing magnitude principle.

Teachers can provide opportunities for practicing the application of the increasing magnitude principle while playing games that involve keeping score. A teacher can have two children compare their scores (represented by two sets of blocks or other markers) and see who won by counting. The teacher could summarize the process by saying, for instance, "Manny has five, but Keisha has one, two, three, four, five, *six*. Six is more than five, because six comes after five when we count."

Figure 2. Sample cardinality chart[57]



To prepare children to mentally compare numbers, teachers can help them master number-after relations (the fourth step in the developmental progression illustrated in Table 3). Everyday situations provide numerous opportunities to incorporate the use of number-after skills. For example, a teacher can say, "Jahael is having a birthday tomorrow; if Jahael is 4 now, how old will he be tomorrow?" or "We just passed Rooms 3 and 4. The next room should be what number?" or "Today is December 4. Tomorrow will be December what?"

Once children have mastered making concrete comparisons using one-to-one object counting and number-after relations, teachers can help them mentally compare neighboring number words (the fifth step in the developmental progression illustrated in Table 3). Teachers may

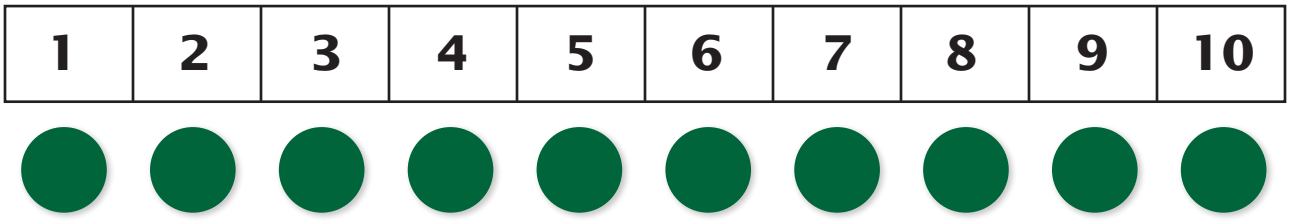
Some children may initially have trouble answering the question "What comes after six?" However, they may be successful if given a running start—counting from "one" up to a number (e.g., "What comes after 'One, two, three, four, five, six'?"). As children master number-after relations, they learn to determine the number after a counting word without using a running start.

find that a number list, or a series of numerals in order, can be used to compare numbers (see Figure 3).[58] Children can see which numbers are "more" or "fewer" based on the numbers' positions on the list. Number lists may be particularly helpful for comparing two collections: by counting with a number list, children can see that numbers earlier and later in the list denote lesser and greater cardinalities and, therefore, indicate smaller and larger quantities. As children practice, these comparisons can be done without the aid of a number list. Transitioning between activities provides a good opportunity to reinforce these types of questions. Children can answer a quick "Which is more?" question before transitioning to the next activity.

As children master the increasing magnitude principle and become comfortable with number-after relations, teachers can demonstrate that a number immediately after another is one more than its predecessor. Children may know, for example, that seven comes after six when we count and that seven is more than six, but they may not realize that seven is exactly one more than six and that each number in the counting sequence is exactly one more than the number before it.

A **number list** is a series of numerals beginning with 1 and ordered by magnitude. Number lists are similar to number lines; however, they do not include 0 and are an easier tool for young children to use when counting and learning numerals.

Figure 3. Sample number list



4. Encourage children to label collections with number words and numerals.

Once children have practiced recognizing, counting, and comparing quantities, teachers can introduce numerals to children as a way to represent a quantity.[59] Sometimes, children may begin to recognize the numerals in the world around them (e.g., on electronic devices, on street signs, or on television) before they are able to count. However, once children have a foundation for understanding number and counting, it may become easier for them to learn about numerals. Teachers can pair numerals with collections of objects around the classroom so that children start to learn, for example, that the numeral 3, three objects, and the spoken word "three" represent the same thing. If teachers use activity centers in their classrooms, they can number those centers with signs that have a numeral, dots representing the numeral, and the number word (e.g., "3, ▪ ▪ ▪, three"). Children who do not yet recognize numerals can use the dots to count and figure out what the numeral indicates. A wide variety of games, such as the memory game *Concentration: Numerals and Dots* (see Example 3), can serve as practice in identifying and reading numerals.

Example 3. The *Concentration: Numerals and Dots* game

Objective

Match numerals with corresponding quantities.

Materials needed:

- One set of twenty cards: ten cards with numerals from 1 to 10 along with the corresponding number of dots, and ten cards with pictures of objects (the numbers of objects corresponding to a numeral 1–10).
- For even more advanced play, once children are proficient at numerals 1–10, teachers can create cards for numerals 11–20.

Directions: Half of the cards have a numeral and dots to represent the amount (e.g., the numeral 3 and three dots) on one side, and the other half have pictures of collections of objects on one side (e.g., three horses, four ducks). The other side of each card is blank. The cards are placed face down, with the numeral cards in one area and the picture cards in another. A player chooses one numeral card and one picture card. If they match, then the player keeps those cards. Play continues until no further matching cards remain. The player with the most cards wins the game.

Early math content areas covered

- Numeral recognition.
- Corresponding quantity.
- If the objects in the pictures on the cards are in different orders, it can help reinforce the idea that appearance does not matter when it comes to number.

Monitoring children's progress and tailoring the activity appropriately

- Play the game with a small group of children, noting each child's progress in practicing and achieving the objectives.
- This game can be played with children who are not familiar with numeracy concepts. Use fewer cards, lower numbers, or cards with dots to scaffold. As children gain proficiency with the concepts, increase the number of cards and the size of the numbers.

Using the activity to increase math talk in the classroom

- Before asking, "How many?" ask, "How can we find out how many?"

5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Using their number knowledge to solve arithmetic problems can give children a context to apply and expand this knowledge and gain confidence in their math ability.[60] Once children can determine the total number of items in a collection by using small-number recognition or counting and can understand the concepts of "more" and "fewer," they can explore the effects of adding and subtracting items from a collection. One way to help children apply their knowledge is to create activities that involve manipulating small

sets of objects.[61] Children can change small collections of objects by combining or removing objects (e.g., adding two blocks to three blocks) and then count to determine "how many" they have in the new collection. As children become more adept, teachers should present more difficult problems with slightly larger numbers. Problem solving can be useful even if children have not completely mastered fundamental number skills, as problem solving may serve as a vehicle for children's learning. Problem solving challenges children to use their math knowledge to answer and explain math-related questions, providing them with an opportunity to strengthen their math skills.

Teachers can use problem-solving tasks across classroom situations so children can see how to apply counting to solve everyday challenges. For example, when children are preparing to play games in small groups, the teacher can ask them to count how many groups there are and use that number to determine how many games to distribute. Once children can consistently use counting to solve simple problems, teachers can ask the class to help find out how many children are in attendance by first asking how many boys there are, then how many girls, and finally how many children in total. Examples with a real-life application for the skill (such as finding out how many children need a snack) are the most helpful to children's learning.[62]

Once children have experience with combining or separating objects in a collection they can see, they can do the same with collections of objects (e.g., pennies) when the final outcome is hidden from view.[63] This arrangement can be in a hiding game that is an extension of the *Basic Hiding* game (see Example 1) or *Hidden Stars* (see Example 2). Teachers can place three or four objects in a line while the children watch. Teachers can then cover the objects (with a cloth or with a box that has an opening on the side) and, while the objects are covered, take one or two additional objects and add them to the objects under the cover. (Alternatively, they can reach beneath the cover to take one or two objects away.) The children see the initial group of objects and the objects being added or taken away, but they do not see the final set of objects. The children must then determine, without looking at the final set of objects, how many are hiding. Children may solve this problem by counting on their fingers or in their heads. After the children give their answer, the teacher can take the cover away, and the children can count to check the answer.

Snack time is also a great opportunity to provide children with authentic comparisons of adding and subtracting or "more" and "fewer." As children receive or eat their snacks, they can count how many items they have. Teachers can also adapt this activity for children of varying skill levels by asking each child different questions, such as "How many will you have after you eat one?" or "How many will you have after your friend gives you one?" Because the number will change, this activity provides good practice for understanding comparisons of more and fewer and combining or removing objects.

Potential roadblocks and solutions

Roadblock 1.1. *I want to provide strong math foundations for my children, but I am not really comfortable with math myself.*

Suggested Approach. Teachers who are not comfortable teaching math can begin by looking for opportunities to teach math in regular activities or familiar situations. They can then design classroom projects that highlight the everyday uses of math. For example, quick counting tasks such as figuring out how many children need a snack, or how many mittens or hats children have, are easy ways to incorporate counting into everyday events. Activities such as setting up a pretend grocery store in the classroom allow children to practice counting food and money. Other examples include community

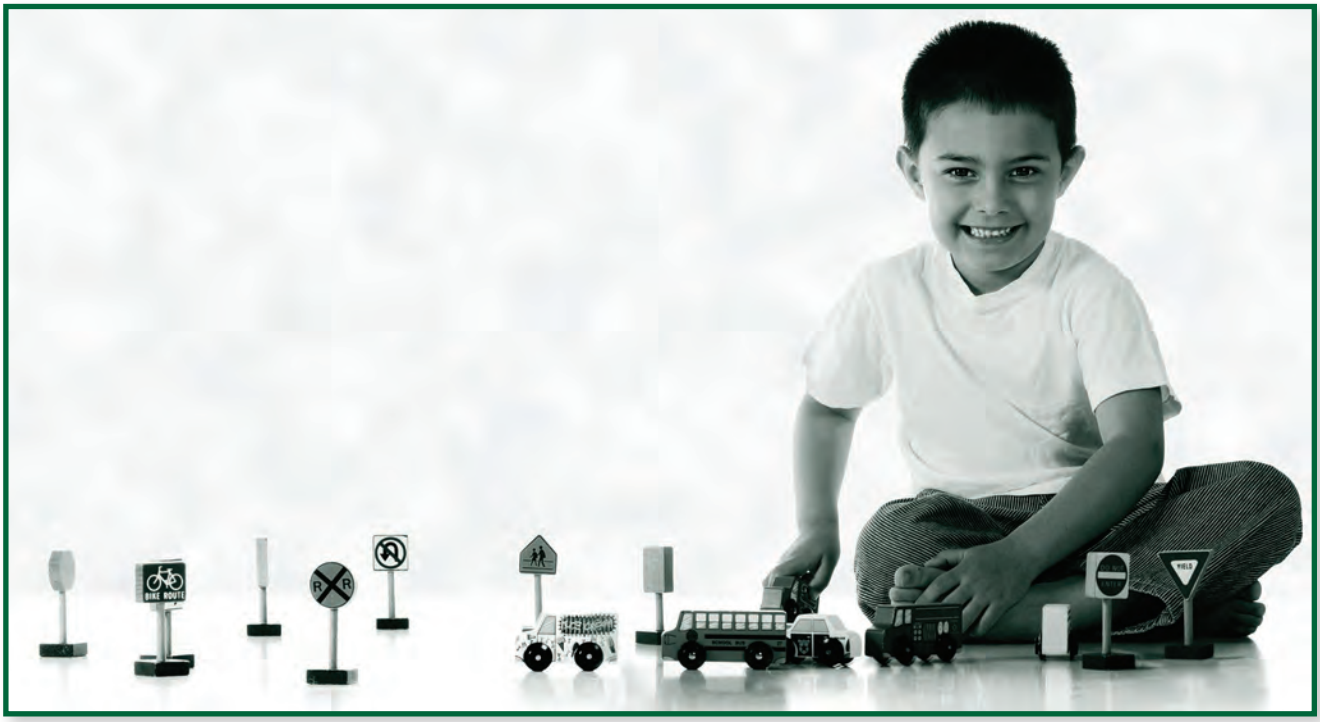
service projects, such as canned-food drives, which can provide opportunities for children to count, sort, label, and organize donations. Sports can also provide children with chances to practice math—for example, measuring the distance for a race on the playground, recording times, and making a chart to display results. Teachers can also consider sharing their own interests with children and highlighting whatever math is involved, such as the measurement involved in cooking or sewing, the geometry involved in woodworking, and so on.

Roadblock 1.2. *Each child in the class is at a different level in the developmental progression I am using to guide instruction.*

Suggested Approach. Teachers can prepare whole-group lessons that target specific concepts and then use small-group activities in which children are grouped with peers who are at a similar level. One group of children can work on activities that are related to a more basic skill (such as counting objects), and another group of children can work on a more advanced activity (such as combining sets of objects and figuring out how many there are in total). Decreasing and increasing the quantity of a collection, using a color-coded die or dice labeled with numerals for playing board games, and increasing complexity of pattern activities while using the same objects are all simple ways to tailor activities. Alternatively, children can be grouped with other children who are at a more proficient level (heterogeneous groups) and can model the skill.

Roadblock 1.3. *A child is stuck at a particular point in the developmental progression.*

Suggested Approach. It may be useful to go back and make sure the child has learned the prerequisites for each step in the progression. Teachers can go back a step and give the child a chance to practice and reinforce skills in a previous level before trying the more challenging level again. It is also important to take into account what concept a child is developmentally ready to learn. Some children may need more practice with a particular skill before moving on to a more complex skill.



Teach geometry, patterns, measurement, and data analysis using a developmental progression.

Children's exposure to math should extend beyond number and operations to include a range of math content areas, including geometry (shapes and space), patterns, measurement, and data analysis.[64] As with Recommendation 1, these math content areas should be taught according to developmental progressions. Learning skills beyond number and operations creates a foundation for future math instruction, and children with strong backgrounds in these areas are more likely to succeed in later grades.[65] For example, early instruction in shapes and measurement lays the groundwork for future learning in geometry, and simple graphing exercises are the foundation for more advanced concepts such as statistics.

When children's understanding extends across a range of math content areas, they have the tools they need to explore and explain their world.[66] They learn that math is everywhere. Geometry is a part of their environment in the form of traffic signs, maps, and buildings. Patterns occur in nature. Measurements help children compare and quantify the things they experience. Collecting and organizing information, such as creating charts to display favorite animals or foods, allows children to find out more about one another.

The steps of this recommendation describe general developmental progressions through the early math content areas of geometry, patterns, measurement, and data analysis. Each component of this recommendation will indicate where to begin within each early math content area and how to progress to more advanced concepts.[67]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expertise and 12 randomized controlled trials[68] and 1 quasi-experimental study[69] that met WWC standards and examined interventions that provided targeted instruction in one or more of the early math content areas of Recommendation 2. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The 13 studies examined interventions that included different combinations of the early math content areas that are the focus of Recommendation 2.

- Ten separate interventions taught young children about geometry.[70] Each of these interventions was tested in at least one of the 12 studies. Positive effects were found for geometry, operations, and general numeracy outcomes, whether the teaching of geometry was part of a broader curriculum or the only component of the intervention. The interventions that taught geometry ranged from early math curricula with multiple units and lessons that focused on geometry,[71] to a curriculum with eight sessions in a four-week period (in addition to regular classroom instruction) that used a story to teach part-whole relations skills.[72]
- Eight interventions taught patterns.[73] These interventions were examined in 10 studies.[74] Six studies reported positive effects in the domains of general numeracy and geometry.[75] One study found positive effects in basic number concepts, operations, and patterns and classification.[76] One study found no discernible effects in operations, and two studies found no discernible effects in operations, general numeracy, and geometry.[77]
- Seven interventions taught measurement.[78] These interventions were examined in nine studies. Positive effects were found in the domains of general numeracy, geometry, and basic number concepts.[79]
- Six interventions taught data analysis.[80] These interventions were examined in eight studies. Six of the studies reported positive effects in the domains of general numeracy and basic number concepts.[81] The remaining two studies reported no discernible effects in the domains of operations, general numeracy, and geometry.[82]

The body of evidence assessed in relation to Recommendation 2 was promising. However, three issues with the evidence prevented the panel from assigning a moderate evidence rating to this recommendation.

First, none of the 13 studies that contributed to the body of evidence for Recommendation 2 evaluated the effectiveness of instruction based on a developmental progression compared to instruction that was not guided by a developmental progression. As a result, the panel could not identify evidence for teaching based on any particular developmental progression. Second, although the research tended to show positive effects, some of these effects may have been driven by factors other than the instruction that was delivered in the four content areas covered by Recommendation 2 and operations. For example, most interventions included practices associated with multiple recommendations in this guide (also known as multi-component interventions).[83] The panel was also concerned about the lack of specific information about how much time was spent on each early math content area in the intervention and comparison groups. Finally, many studies reported on outcomes that were not directly aligned with the early math content areas included in this recommendation.

Together, these three limitations resulted in the panel not being able to claim with certainty that the effects seen were due solely to targeted instruction in the early math content areas of geometry, patterns, measurement, and data analysis. Nevertheless, the panel believes the positive effects found for interventions based on a developmental progression when compared to instruction that does not appear to be based on a developmental progression support their recommendation to use a developmental progression to guide instruction. When combined with the positive effects found for interventions that included targeted instruction in geometry, patterns, measurement, and data analysis, the panel believes the studies generally support this recommendation, despite the limitations to the body of evidence.

The panel identified four suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. Help children recognize, name, and compare shapes, and then teach them to combine and separate shapes.

Teachers should encourage children to recognize and identify shapes in their surrounding environment.[84] Children may find shapes in their drawings, bring an object from home that illustrates a particular shape, or locate shapes in the classroom.

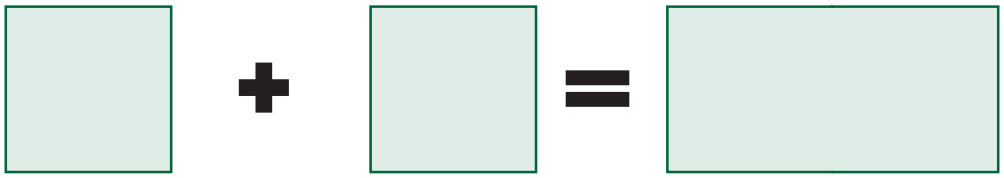
When children can confidently recognize shapes, teachers should then provide opportunities for children to name the critical attributes of shapes using standard geometric terms. A critical attribute of a shape is a characteristic shared by all examples of that shape. For example, all rectangles have four sides, and the opposite sides are equal and parallel. Although many rectangles have two long sides and two short sides, some do not. Therefore, having two long sides and two short sides is not a critical attribute of a rectangle. Squares share all the critical attributes of a rectangle but have the additional critical attribute of four equal sides.

Teachers should provide examples and non-examples of shapes so children can learn to categorize them.[85] A non-example of a shape lacks one or more critical attributes that define the shape. For instance, a long, thin rectangle is a non-example of a square because all the sides are not equal; a diamond (rhombus) is a non-example of a triangle because it has four sides instead of three. These and other examples and non-examples allow children to make distinctions about the basic features of shapes, paving the way for learning about relationships among shapes.

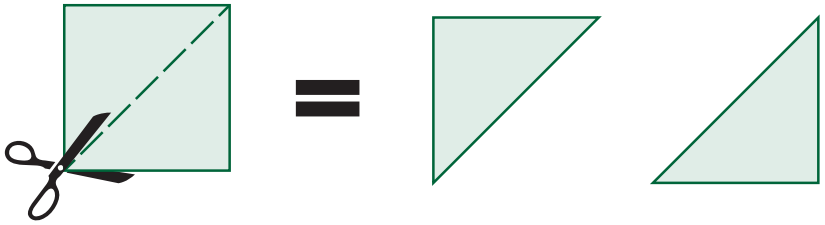
Once children are comfortable recognizing and comparing shapes, teachers should encourage children to explore how shapes can be combined and separated to form new shapes.[86] For example, two identical squares can be combined to form a rectangle, and a square can be cut along the diagonal to form two triangles or across the middle to form two rectangles, as shown in Figure 4.

Figure 4. Combining and separating shapes

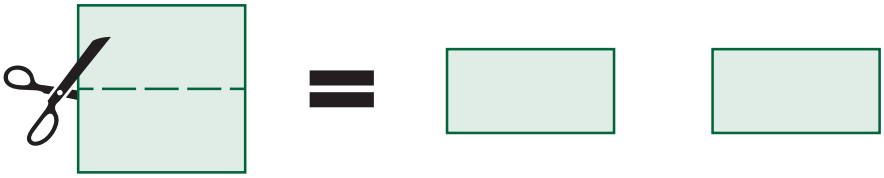
Two identical squares can be combined to form a rectangle.



A square can be cut along the diagonal to form two triangles.



A square can be cut across the middle to form two rectangles.



Exercises such as the *Shapes* game, outlined in Example 4, reinforce the properties of shapes and the spatial relations between them. When children manipulate shapes, they learn that changes in orientation do not affect the critical attributes of the shape.[87] They can also learn about spatial relationships between objects, such as "in," "on," "under," "beside," "above," or "below."

Example 4. The *Shapes* game

Objective

Identify and discuss attributes of various shapes and how to manipulate shapes to fit inside a larger field.

Materials needed:

- A large piece of poster board with a large shape drawn on it
- Various (precut) foam or plastic geometric shapes

Directions: Children draw from a basket or bag containing a variety of small shapes to put on the large shape drawn on a piece of poster board. The children take turns choosing a small shape from the basket and then identifying it, describing it, and placing it on top of the large shape. The group works together to fit as many small shapes as possible within the borders of the large shape without overlapping any of the shapes. When children have finished filling the large shape, they can count how many of each small shape they used and how many shapes were used in total. For subsequent games, the children can try to choose and place shapes strategically so the group can fit more small shapes inside the large shape. Teachers can tailor the *Shapes* game for use with the whole class, a small group, or individual children.

Early math content areas covered

- Geometry (shapes and attributes of shapes)

Monitoring children's progress and tailoring the activity appropriately

- Observe and note each child's ability to identify a shape and describe its attributes (number of sides, angles, and so on).
- Note children's ability to manipulate and place a shape strategically so the maximum number of shapes can be used.
- For inexperienced children, use only basic shapes (square, circle, triangle, and rectangle). As children become more proficient with the activity, increase the complexity of the shapes.

Integrating the activity into other parts of the day

- Blocks offer an opportunity for children to strategically manipulate and combine shapes to create other shapes and build more complex structures.

Using the activity to increase math talk in the classroom

- Talk about and describe shapes in the environment inside and outside the classroom.

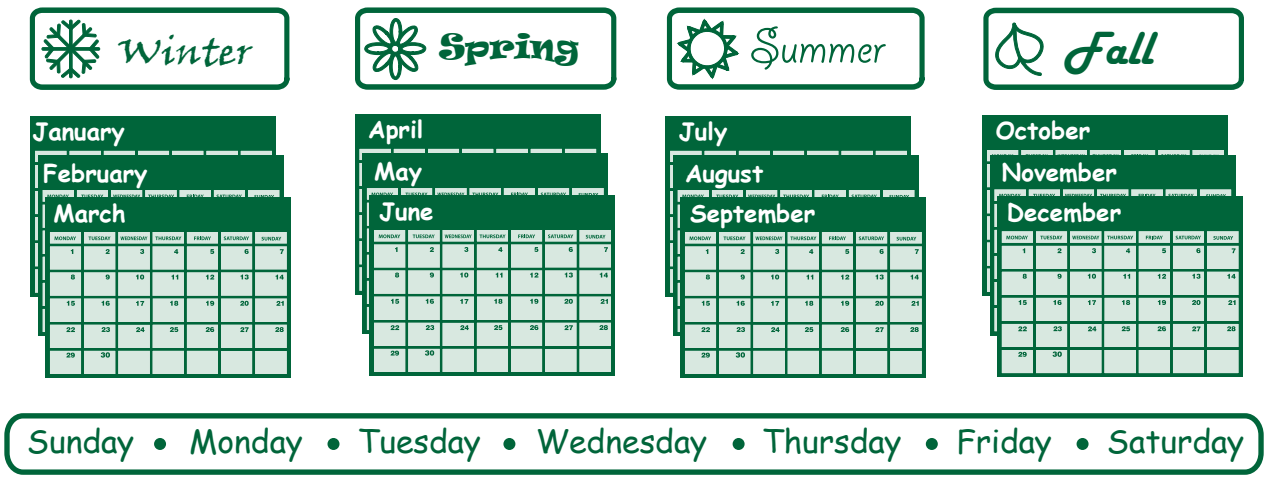
2. Encourage children to look for and identify patterns, then teach them to extend, correct, and create patterns.

Pattern instruction should begin by encouraging children to experiment with basic repeating patterns. For example, teachers can select a child to establish the pattern in which the rest of the class will line up for an activity (e.g., boy, girl, boy, girl, boy, girl). As children become familiar with simple patterns, they can experiment with more complex ones (e.g., boy, boy, girl, girl, boy, boy, girl, girl, boy, boy, girl, girl, as pictured in Figure 5).

Figure 5. Moving from simple to complex patterns

Teachers can encourage children to notice the patterns in the world around them, such as stripes on clothing, shapes and designs in rugs, planks in a wooden floor, or bricks on the sides of buildings.[88] Teachers can also describe the repetitive nature of the days of the week (Sundays are always followed by Mondays) and the number of months in a season, as displayed in Figure 6.

Figure 6. The repetitive nature of the calendar



Once children have become familiar with the nature of patterns, they should learn to predict what will happen next in a pattern, based on what has happened so far.[89] Children can use manipulatives, such as colored beads, to experiment with how patterns work. For example, teachers can create a string of alternating red and blue beads, and then instruct children to select the next bead in the string based on the current pattern. Teachers can also create errors in the previous pattern, such as two blue beads following a red bead,

and ask children to correct the errors. As children's understanding grows, teachers can provide opportunities for children to create patterns based on a set of instructions. For example, teachers could present the beads and strings to children and ask them to make a pattern in which two red beads follow every blue bead. Teachers can add complexity to the activities by introducing additional colors or other categories of beads based on size (big or small) or shape (round or square). Teachers can also encourage children to experiment and create patterns on their own, as outlined in Example 5.

Example 5. Creating and extending patterns

Objective

Recognize and create patterns of increasing complexity.

Materials needed:

- Short strings with a knot or fastener tied at one end
- Colored beads

Directions: Distribute short strings and handfuls of colored beads to the children. Create an example of a pattern, such as a red bead followed by a blue bead followed by another red bead. First, ask the children to recreate the existing pattern. Next, ask the children to predict which color will come next in the pattern. As the children's understanding grows, create patterns with deliberate errors (for example, following the blue bead with a second blue bead in the exercise above) and then ask the children to identify incorrect sequences. Finally, instruct the children to create patterns on their own. Teachers can tailor this activity for use with the whole class, a small group, or individual children.

Early math content areas covered

- Patterns

Monitoring children's progress and tailoring the activity appropriately

- Vary the number of beads to determine whether children are ready to use larger sets.
- If a child has difficulty, repeat the pattern several times in the same string of beads (e.g., red, blue, red, blue, red, blue). If the child grasps the exercise quickly, use more complicated patterns (e.g., red, blue, red, blue, blue, red, blue, blue, blue).

Integrating the activity into other parts of the day

- Adapt the exercise to include patterns children find in the world around them. For example, encourage children to look for patterns in the tiles on the classroom floor (square tiles and rectangular tiles), the bricks on the outside of the school (big bricks and small bricks), the clothing they wear (stripes, plaids, and other designs), or music they hear (verses and choruses).

Using the activity to increase math talk in the classroom

- Ask children to create patterns using themselves when lining up, and emphasize that a pattern is a repeating sequence.
- Blocks can provide children with an opportunity to create patterns while building structures.

3. Promote children's understanding of measurement by teaching them to make direct comparisons and to use both informal or nonstandard (e.g., the child's hand or foot) and formal or standard (e.g., a ruler) units and tools.

Teachers should show children how to compare objects for the purpose of sorting, arranging, and classifying them.[90] Teachers can help children understand what it means to compare the characteristics of two objects and identify similarities and differences. For example, as children's understanding of comparisons develops, children can begin to compare the lengths of two pieces of string to determine which is shorter or longer. Teachers can expand on this concept by demonstrating how to arrange a collection of pieces of string from shortest to longest. When making comparisons, teachers should reinforce measurement vocabulary words that describe the characteristics of the objects and the differences between them. Table 5 provides examples of vocabulary words associated with different types of measurement.

Once children have become comfortable making direct comparisons between and among objects, teachers can provide children with opportunities to measure objects using nonstandard tools and informal units, such as children's own hands and feet, or classroom items such as pencils, blocks, or books. After children learn to assign numerical values to the objects they are measuring with nonstandard tools (such as measuring the width of a table by counting how many "hands across" it is), they should be introduced to the concept of standard units of measurement (e.g., inches, feet, ounces, or pounds) as well as measurement tools (e.g., rulers and scales). Practice with these concepts can help lay the foundation for learning formal measurement vocabulary, tools, and techniques in later grades.[91]

By first using nonstandard measurement and then progressing to standard ways of measuring, children will discover that nonstandard measurements can vary, but standard measurements do not. For example, children could measure something familiar, such as the distance from the door to the writing center or the distance from the classroom

Table 5. Examples of vocabulary words for types of measurement

Type of Measurement	Examples of Vocabulary Words
Length 	long, longer, longest; short, shorter, shortest
Size 	small, smaller, smallest; big, bigger, biggest
Temperature 	warm, warmer, warmest; cold, colder, coldest
Time 	early, earlier, earliest; late, later, latest
Weight 	heavy, heavier, heaviest; light, lighter, lightest

to the restroom, by counting the number of steps between the two locations. Teachers could emphasize that children's measurements may vary depending on the size of the steps they take. Once children have learned to assign numerical values and use measurement vocabulary and tools, they can measure the distance in standard feet and inches using rulers and yardsticks.

Other opportunities for practicing measurement concepts include monitoring growth in height and weight, changes in temperature ("Today is warmer than yesterday") through different seasons, and differences in time ("We eat breakfast in the morning, and we eat dinner at night"). Children will learn that thermometers, scales, and rulers produce more consistent measurements than nonstandard tools. Understanding the numerical values associated with measurement will then help children make comparisons between objects. Children can utilize their existing knowledge of number to determine that an object with a length of 10 inches is longer than an object with a length of 5 inches because ten is more than five.

4. Help children collect and organize information, and then teach them to represent that information graphically.

Teachers should provide children with opportunities to count and sort familiar items to introduce them to the concept of organizing and displaying information.[92] This information can take the form of tangible objects, such as toys or blocks, or abstract concepts, such as characteristics (e.g., which children are 4 years old and which children are 5 years old) or preferences (e.g., favorite snacks, colors, or animals). The goal of such exercises is to demonstrate both the characteristics that distinguish the items and the total number in each set relative to other sets. For example, teachers could introduce sorting exercises when children are cleaning up and putting away toys. For children interested in building, teachers could encourage recording the number of different types of blocks. For children interested in drawing, teachers could encourage sorting, counting, and recording the number of crayons versus markers versus colored pencils.

Once children are familiar with sorting and organizing the information they have collected, they should learn to represent their information visually.[93] Graphs allow children to summarize what they have learned, and graphing provides an opportunity for children to share and discuss their findings.[94] Teachers can begin by introducing simple tallies and picture graphs to children, then teaching children to interpret the meaning of these graphs. Teachers can eventually move on to more complex graphs to illustrate changes in children's height or weight or to describe different characteristics of children in the class (e.g., gender, favorite color, clothing, or hair color). Example 6 describes a game in which children sort and discuss information with the class.

Example 6. The *Favorites* game

Objective

Have children practice sorting and grouping.

Materials needed:

- Signs for each sorting category, located in different areas of the classroom. In this example, children are sorting based on their favorite food.

Directions: Create a sign for each food, and place the signs in different areas of the classroom. Then, ask each child to share his or her favorite food with the class. Have the children find and stand near the sign that designates their favorite food. Once every child has joined a group, ask the children which food is the most common and which is least common.

Early math content areas covered

- Organizing and presenting information
- Number and counting

Monitoring children's progress and tailoring the activity appropriately

- Note each child's ability to name his or her favorite food, select the appropriate group, and answer questions about the information gathered.

Integrating the activity into other parts of the day

- Transition children by favorite food (e.g., "All the children who like apples can line up").

Using the activity to increase math talk in the classroom

- When children have sorted themselves, ask comparison questions such as "Which group has the larger/smaller amount?"

Potential roadblocks and solutions

Roadblock 2.1. *It is challenging enough to cover everything I need to cover in a day without having to think about four more early math content areas.*

Suggested Approach. Teachers may be able to find opportunities to cover more than one early math content area (number and operations, geometry, patterns, measurement, and data analysis) in the context of a lesson. For example, children can bring a collection of objects from home or find a collection of objects during recess. Children can first count the items in the collection and then arrange them in a pattern. Teachers can encourage children to identify any shapes in the collection and to name the critical attributes of those shapes. Children can be prompted to arrange the items according to characteristics such as size, length, weight, and so on. Finally, teachers can instruct children to sort their collections, compare the groups, and represent the information in a simple graph to identify which groups have more, fewer, or the same number of items. Addressing multiple math content areas within one activity might make it easier for teachers to cover all of the material assigned to that day. Another approach is to develop math games that can be played during transitions and down time that both help with classroom management and reinforce math concepts, particularly ideas that children have found challenging that week. For example, "I spy" games can be played anywhere and can be used to practice identifying shapes or patterns.

Roadblock 2.2. *Some children are struggling with basic vocabulary skills or are being exposed to English for the first time.*

Suggested Approach. Teachers can link visual representations of the most important vocabulary and concepts for geometry, patterns, measurement, and data analysis with terms in the child's home language, as well as in English, particularly when multiple children in the classroom speak the same language.[95] Teachers can help English-speaking children learn to count in their classmates' native languages to learn about each other. Songs and fingerplays are helpful tools for learning new words and math concepts. Using math manipulatives and inviting children to arrange materials or draw to show their answers can also help bridge the language gap.

Recommendation 3

Use progress monitoring to ensure that math instruction builds on what each child knows.

Evidence from studies of several math curricula suggests that preschool, prekindergarten, and kindergarten children are most likely to gain math knowledge when they are frequently exposed to targeted, purposeful, and meaningful math instruction.[96] Progress monitoring can be a useful way to ensure that children are receiving this type of instruction.

When developmental progressions (as described in Recommendations 1 and 2) are combined with progress monitoring, teachers can adapt lessons to a child's growing math knowledge. Effective instruction targets a child's developmental level (i.e., the child's skill level based on a developmental progression) and helps the child achieve the next level in the progression.[97] Connecting the information that is currently being taught to what children already know facilitates learning. By continually monitoring a child's progress, teachers can gather the information they need to match lessons to an individual child's knowledge level. Children develop knowledge at different times and at different paces.[98] Deliberately incorporating these individual differences into lesson planning by monitoring progress and tailoring instruction can help ensure that all children are encouraged to learn math concepts and skills that are appropriately challenging and just beyond their current level of understanding.[99]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expert opinion and 11 randomized controlled trials[100] and 1 quasi-experimental study[101] that met WWC standards and examined interventions that included at least one component of Recommendation 3. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The 12 studies examined curricula that included regular, short assessments during lessons. These assessments may have been informal, computer-based, or supported by rubrics to be used by the teacher during small-group instruction. Two interventions that included regular, short assessments were examined in six studies. Four of the six studies examined an intervention that included supports for assessments. On average, children who participated in the intervention scored higher on math outcomes than did children in the comparison condition.[102] Two of the six studies examined a number sense curriculum that included regular informal assessments to support the tailoring of review sessions. Once again, children who participated in the intervention tended to score higher on math outcomes than children in the comparison condition.[103]

Additionally, some curricula included "upward" and "downward" extensions of activities to support teachers in tailoring their instruction. The study examining *Pre-K Mathematics*, which provides both assessment tools and extension activities, found that children who participated in *Pre-K Mathematics* scored higher on average on children's general numeracy as measured by the Child Math Assessment (CMA) than children participating in the school's regular math instruction which may not have provided assessment tools and extension activities.[104]

The panel concluded that the body of evidence assessed in relation to Recommendation 3 was promising. However, it was not sufficient to warrant a moderate evidence rating as the panel was unable to definitively attribute the effects in the studies to the strategies included in Recommendation 3 due to two characteristics of the studies. First, the interventions examined in the studies were multi-component interventions that included strategies related to Recommendation 3 and other recommendations in the guide.[105] As such, it was difficult to determine whether the use of progress monitoring alone, or in combination with other program components, was responsible for the effects seen in math achievement. It is also possible that progress monitoring had no effect, and other components (or practices) were responsible for effects observed. Second, in most studies, the difference in the amount and type of progress monitoring the intervention and comparison groups received was not always specified,[106] and thus was not considered a direct test of a key component of the recommendation. Based on its expertise and the effects of interventions that include progress monitoring, the panel believes the studies generally support this recommendation despite the limitations to the body of evidence.

The panel identified three suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. Use introductory activities, observations, and assessments to determine each child's existing math knowledge, or the level of understanding or skill he or she has reached on a developmental progression.

When employing progress monitoring, teachers should first gather specific information about each child's skill level to determine where to focus instruction. The panel suggests three primary methods of determining children's level of math understanding: introductory activities, observation, and formal assessments.

- **Introductory activities** involve presenting a new concept to determine how much of the activity children are able to do independently. For example, teachers can begin a small-group lesson on shapes by giving each child a bag of small shapes, including a triangle, a square, a rectangle, and other assorted shapes. If possible,

these shapes should differ in size and color for each child. After presenting a lesson on the different shapes, the teacher could ask younger children to name and compare the shapes in their bags, inquiring whether there are fewer blue circles or green triangles in the bag, which rectangle is the longest, or which circle is the smallest. Teachers could challenge older children to remove a shape from the bag—a rectangle, for example—and to tell the group how they know it is a rectangle. This kind of introductory activity can provide an opportunity for the teacher to assess a child's ability to sort shapes with similar features and classify them using math vocabulary.

- **Observation** involves using a math activity that addresses a specific skill and watching how children try to complete or solve the task. Often, watching children trying to solve a problem provides information about what knowledge they have and what knowledge they lack (see the *Monitoring children's progress and tailoring the activity appropriately* section of each Example for more progress-monitoring suggestions). Additionally, teachers can discover what children understand by asking them questions that require the children to think out loud and describe their problem-solving processes. Teachers can use these techniques to determine whether children are ready to move on to a more advanced concept or need more practice.
- **Formal assessments** typically occur at designated times of the year and can be standardized tests or other assessments that may not be chosen or administered by the teacher. Such tests can serve as screening and planning tools if used before or during instruction. If teachers receive feedback on children's performance on these assessments, they can use the information to plan activities and lessons. In addition to looking at total scores, it can often be useful to examine how children answer particular questions. It may be clear from some test sections that children are struggling with particular concepts, such as number recognition or counting. This information can help teachers direct their instruction to particular goals.

2. Tailor instruction to each child's needs, and relate new ideas to his or her existing knowledge.

Teachers should continually monitor a child's learning by employing a combination of strategies from the first step in this recommendation and should then use that information to design instructional activities.[107] Once teachers have information about a child's skill level, they can use a developmental progression to determine what the child should learn next and then can choose activities that are at or slightly above the children's level of understanding. For example, once a child can use small-number recognition to compare small collections, he or she can use meaningful object counting to determine the larger of two collections (for more details on a developmental progression for number and operations, see Table 3). Activities that are only slightly above the child's level of understanding can help ensure that the child does not feel frustrated by an activity that is too difficult. For example, knowing how many objects a child can successfully count in a set allows the teacher to gradually increase the number of objects so that the child can practice counting larger sets.

When tailoring instruction to individual students, the goal is not only to build on a child's existing math knowledge, but also to connect instruction to his or her interests in a variety of content areas. Relating new skills to children's existing understanding and experiences can help build knowledge. For example, if children have a particular interest

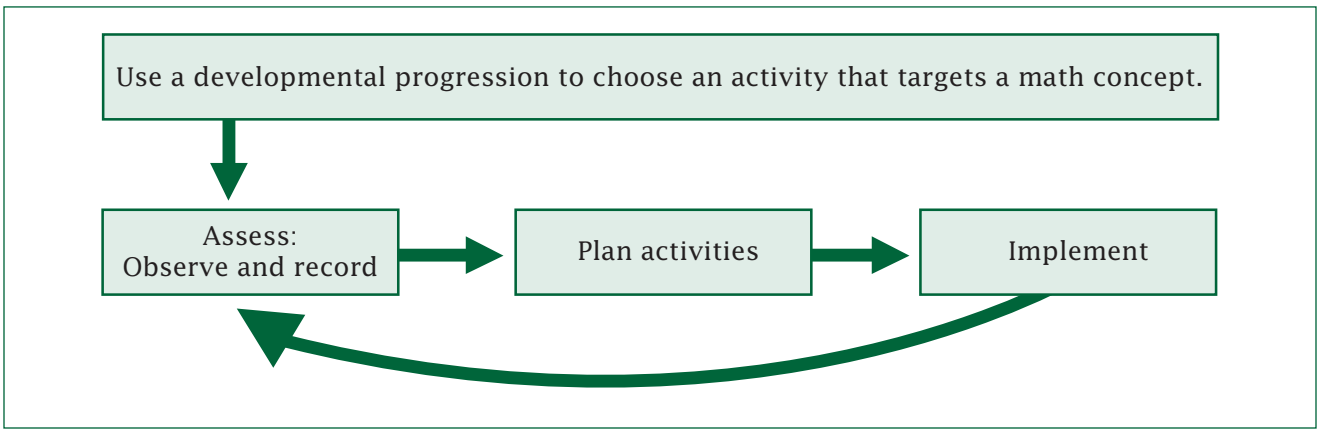
in music, teachers can design math activities that involve musical instruments. Children can determine how many instruments they need for everyone to play together or how many sticks are needed to play all the drums; they can count, sort, and compare different sets of instruments (how many drums, how many wind instruments, etc.); they can count along with musical beats, claps, or marching; and they can create musical patterns (e.g., one drum beat, two claps, one drum beat, two claps). By engaging children in activities that are interesting and applicable to their daily lives, children can connect skills across different activities and content areas.

Small-group activities can be a useful way of adapting instruction when children in a class are at different developmental levels and abilities. For example, using small-group time to play board games is one way that children of different abilities can make connections among their math skills. As the children learn more, teachers can adjust the game based on the children's level of understanding. For example, teachers can tailor a board game to different developmental levels by customizing the spinner. The teacher can first use a color-coded spinner that matches colored spaces on the board, so that children can use a spinner without numbers. The teacher can then introduce a spinner that has both dots (representing the number of spaces to be moved) and numerals. These types of materials can be changed throughout the year: early in the year, children can rely on color; later, they can count the dots on the spinner; and finally, they can use numerals to play the game. For more examples of using games to teach math concepts and skills, see Recommendation 5.

3. Assess, record, and monitor each child's progress so that instructional goals and methods can be adjusted as needed.

It is important to continually monitor progress so that children can be consistently engaged in activities that are neither too far below their level (and therefore not interesting) nor too far above it (and therefore frustrating). Progress monitoring also allows teachers to plan what children should learn next. Example 7 contains a model of the flow of progress monitoring. In this model, a teacher focuses small-group instruction on counting small collections. The teacher observes and records the children's progress using the checklist in Example 8. Looking at the largest set counted successfully and the type of errors made, the teacher can plan different activities for the two children, Sarah and Bill. Sarah should continue counting small collections, while Bill is able to move on to comparing magnitudes of collections. The teacher should also plan to reassess Sarah and Bill, repeating the ongoing process of progress monitoring.

Example 7. The flow of progress monitoring



While engaging in progress monitoring, teachers may want to keep track of their observations, as shown in Example 8. While children are involved in a math activity, the teacher can observe and quickly note what each child can and cannot do. By keeping a record of children's skill progression, teachers can more easily determine where a child may need extra help or what activities the child can do particularly well. For example, a teacher can observe a child counting objects to assess whether the child can successfully count with one-to-one correspondence. If the teacher notices a child making a coordination or sequencing error, the teacher can note the type of error to help determine which activities the child should work on next to practice this skill. (See Table 4 for common counting errors.)

Example 8. Progress-monitoring checklist

Activity: How many stars are there? (Child is asked to point and count "how many stars.")

Child	Date	Activity	Largest Set Counted Successfully	Types of Errors Made
Sarah	September	counting stars	5	skips "six" when counting
Bill	September	counting stars	10	sometimes double-counts a star

Potential roadblocks and solutions

Roadblock 3.1. *How can I maintain order in the classroom when breaking the class into small groups?*

Suggested Approach. When children are in small groups, classroom behavior can sometimes become chaotic and noisy. There are three things to think about when forming small groups. First, group children strategically to avoid social conflicts. If children of mixed abilities are working together in groups, ensure there is the right mix of ability levels. Second, develop activities that build on children's interests. Using small groups enables teachers to present more challenging activities to some children so that they do not become bored. Finally, plan adult assistance to facilitate independent and adult-supported learning for all groups. One strategy for managing groups is to use round-robin learning centers.[108] While one group is meeting with the teacher, other groups are productively engaged in different learning centers. There should be one center for each group not meeting with the teacher. The teacher then is free to focus on one small group at a time.

Roadblock 3.2. *I am already required to give standardized assessments. Can I use my existing assessments to tailor instruction?*

Suggested Approach. Teachers can review the assessments to find questions that apply to the particular skill they would like to target. Then, they can use those questions to gauge where children are and at which level to target activities. If teachers receive feedback on how children in their classroom are performing on a standardized assessment, they can fit this feedback in with the developmental progressions to determine which areas need more focus and when children can move on to higher-level skills. If there are assistant teachers, aides, or other adults available in the classroom, teachers can ask to them to share in observing children and keeping brief checklists of children's ability levels.

Roadblock 3.3. *What if I do not have required assessments, or the assessments do not fit well with the skills that are targeted in the developmental progression?*

Suggested Approach. Teachers can use a developmental progression to develop an activity that will provide information about the child's skill level. For example, teachers can develop a checklist of numerals from 1 to 20 and use magnetic numerals or a numeral bingo game to assess the child's ability to recognize numerals. Teachers can generate checklists for counting collections, naming shapes, identifying patterns, sorting, and many other math skills.

Representations are objects, actions, words, pictures, or symbols that stand for ideas.

Teach children to view and describe their world mathematically.

Teachers can encourage children to look for opportunities to describe math ideas in the world around them, gradually moving from informal representations and language to formal representations and math vocabulary as children's understanding grows.[109] By exploring their environment and interacting with manipulatives, children can begin to apply their math knowledge.[110] At first, children should use informal tools such as their fingers, tally marks, or other concrete objects to represent math ideas. For example, children can be encouraged to use blocks to model and solve simple addition problems (e.g., "If I have two blocks, and I add three more, how many blocks do I have?"). Once children are comfortable using math informally, teachers can help them link their informal knowledge to more abstract math concepts, formal math vocabulary, and formal representations such as math symbols.[111]

If children hear math vocabulary in context and then practice using it, they may be better able to understand the underlying math concepts.[112] The panel believes there is evidence of a positive relationship between math-related talk and children's math knowledge.[113] As one part of math-related talk, teachers can use open-ended questions to prompt children to think about how to describe their ideas mathematically and to increase the amount of math-related dialog in the classroom. If a child can describe his or her method for solving a problem to someone else and then hear other children describe their approach to a problem, all the children may learn to apply their math knowledge in new ways.[114] Teachers can reinforce this idea by encouraging children to look for opportunities to use their developing math skills throughout the school day.

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 14 randomized controlled trials[115] and 2 quasi-experimental studies[116] that met WWC standards and examined the effects of interventions designed to help children view and describe their world mathematically. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

Some interventions provided specific math vocabulary words[117] and suggestions for stories,[118] songs, or questions[119] that supported children in learning to view and describe their world mathematically. Studies examining these interventions found positive effects in the general numeracy, basic number concepts,

and geometry domains.[120] In two studies, math conversation, whether with a peer or an adult, resulted in higher math achievement.[121]

The panel concluded that the body of evidence assessed in relation to Recommendation 4 was promising. However, it was not sufficient to warrant a moderate evidence rating as the panel could not attribute the effects solely to Recommendation 4 for two reasons. First, the examined interventions were multi-component interventions incorporating elements of other recommendations in the guide.[122] Second, in some studies there was a lack of clarity regarding the instruction the intervention and comparison groups received.[123] Based on its expertise and the effects of interventions that include efforts to teach children to view and describe their world mathematically, the panel believes the studies generally support this recommendation despite the limitations to the body of evidence.

The panel identified four suggestions for how to carry out this recommendation.

How to carry out this recommendation

1. Encourage children to use informal methods to represent math concepts, processes, and solutions.

Math instruction for young children should begin with informal representations of math ideas.[124] Initially, teachers should link math ideas to familiar experiences, terms, or analogies, resisting the urge to use more formal methods until children have a conceptual foundation for understanding them.[125] For example, teachers should use terms that represent children's informal understanding of addition, such as "more" and "all together," as opposed to the more formal, symbolic representation. An example of informal understanding might be "Bill had three carrots, and his mother gave him one *more*. How many carrots does Bill have *all together* now?" This phrasing is in contrast to formal representations, such as "Three plus one equals what?" or "3 + 1 = ?" Table 6 provides examples of how to teach informal representations of math concepts.

Table 6. Using informal representations

Concept	Informal Representation	Teaching the Concept
whole number	"three"	Collections of blocks, dots, tally marks, fingers, or other countable objects can represent numerals. For example, when playing a game, use blocks to represent children's scores so everyone can track each player's score.
equal	"same number as" or "same as"	Provide opportunities for children to begin to recognize that collections that have the same number when counted are equal. For example, a collection of four plates is the same number as a collection of four cups.
unequal	"more than" or "fewer than"	Point out that a collection is more (or fewer) than another if it requires a longer (or shorter) count. For example, seven is more than six because it requires counting beyond six.
addition	"and" or "more"	Start with a collection and add more items to make it larger. For example, start with three crayons and add one more. Then ask, "How many?"
subtraction	"take away" or "fewer"	Start with a collection and take away some items to make it smaller. For example, start with three crayons and take away one. Then ask, "How many?"

2. Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences.

Once children are comfortable using informal methods and representations to describe math ideas, teachers can introduce math vocabulary and formal representations. Teachers should explicitly teach children math words so they have the vocabulary needed to connect their informal knowledge to formal terms.[126] Teachers can start with informal vocabulary and then connect these familiar terms to formal terms. For example, teachers might begin with the informal phrase "take away" and then later explain that "subtract" has the same meaning.

Teachers can then use this math vocabulary when speaking to children throughout the day. Vocabulary that is used during math instruction does not need to be restricted only to math activities. For example, words such as "more" and "fewer" can be emphasized throughout many different topics and activities. Math conversations can happen spontaneously as teachers comment about natural occurrences that involve number or other math concepts. For example, teachers can make a comment about which child is standing "first" in line or which child has "more" or "fewer" objects than another child. As another example, while the child is drawing a picture of his or her family, a discussion could focus on the "number" of family members and who is "older" or "younger."

Just as children learn to link math vocabulary to their informal knowledge, they should also learn to connect formal representations to their informal math knowledge. Linking formal representations to informal concepts and representations enables children to understand and more readily learn formal terms, symbols (e.g., + or –), definitions, and procedures.[127] For example, teachers can connect numerals to both quantities (e.g., a collection of five buttons) and verbal representations (e.g., the word "five").[128] Table 7 provides examples of lessons for linking familiar concepts to formal symbols.

Table 7. Linking familiar concepts to formal symbols

Symbol	Concept	Lesson
numerals	counting	Have children count and record the number of children in attendance each day.
+ , –	operations	Have children solve problems involving adding or subtracting with leaves collected from the playground.
=	equal	Show the class four pennies. Next, show three pennies, verbally label them ("I have one, two, three pennies"), and put them in a can. Then, show one more penny, verbally label it ("I have one more penny"), and put it in the can. Ask the class, "Are three pennies and one more penny the same number as four pennies?"
< , >	unequal	Show the class five pennies, verbally label them, and put them in a can. Next, show four pennies, verbally label them, and put them in a different can. Ask the class, "Which can has more? Which can has fewer?"

3. Use open-ended questions to prompt children to apply their math knowledge.

Open-ended questions can help children to develop cognitive and language skills. They prompt children to think through their actions, describe their thoughts, and learn from one another. Questions that begin with "what," "why," or "how" can encourage children to use math vocabulary to explain what they have learned. Teachers should ask questions that require children to use math-related terms to describe something. For example, asking, "How can we find out (how many children are here today, how much snack we need, etc.)?" gives children the opportunity to communicate about a math strategy and then to practice that strategy. The questions can be tailored to current math objectives. See Table 8 for examples of questions teachers can ask that are related to the math content areas.

When asking open-ended questions, teachers can employ techniques to encourage math-related conversation. First, before calling on a child, teachers might allow enough time for more than just a few children to think of an answer. When in groups, one child can help another child come up with an answer. Rather than saying "yes" or "no" quickly, teachers can allow multiple possibilities to be discussed. For example, a teacher can show the entire class a picture of a mother and a daughter holding hands, waiting for the school bus. The teacher can ask "How are these two people different?" One child may answer, "The mother is bigger than the daughter." Another child may answer, "The mom is wearing stripes and the daughter is wearing dots." Although the teacher should ultimately focus on the correct answers that best fit a math context, he or she should acknowledge that there are multiple correct responses.

Table 8. Examples of open-ended questions

How are these the same/different?
What can you use (in the block area) to make a pattern?
What patterns do you see (on the seashells in the science center)?
How could we change this pattern to make a new one?
How can we find out who is taller or shorter?
What can we use to find out…?
What can we do to find out who has more/fewer?
How else can you show it?
How does it show what we know?

4. Encourage children to recognize and talk about math in everyday situations.

Teachers can encourage math thought and conversation by asking children for their help with problems that arise throughout the day.[129] For example, a teacher might say, "I have to figure out how many cups we are going to need for the birthday party. Can you help me? How should we do that?"

Once children solve the problem, teachers can have them describe their method by asking a sequence of questions that prompts the children to share the solution and the strategies used to reach the solution. For example, if the problem involves how many orange slices are needed for snack time, the teacher could ask the children for an answer. Then, the teacher could say, "How did you figure that out? What did you do first? Then what did you do?" During small-group time, the teacher and children could have a more formal discussion about the steps used to solve the problem.

After a child shares his or her solution, the teacher might repeat the problem-solving steps back to the child in sequence to continue the math talk. For example, the teacher could say, "Oh, I see, first you counted how many children were here. Then you thought about how many orange slices each child might eat." To continue the conversation even further, the teacher could ask the group, "Is there another way you can do that?" or "How else could we do this?"

When children are given explicit math problems to solve, it can be helpful for them to talk through their problem-solving process.[130] For example, in an extension of the *Basic Hiding* game in Recommendation 1, when a child successfully tells how many objects are hidden, the teacher can ask the child to describe how he or she knew how many there were. It is important to keep children's developmental levels in mind. At first, many children may not be able to describe their problem-solving process. Teachers can aid children by talking through their own problem-solving strategies out loud, demonstrating for the children how to use math vocabulary when describing their thought processes. As teachers help them with the math conversation and emphasize the math vocabulary (e.g., "There were five blocks, and then I added three more blocks"), teachers can help children begin to develop the skills they need to communicate about the problem solving that they or their peers are doing.

Potential roadblocks and solutions

Roadblock 4.1. *I'm not sure what types of open-ended questions are most effective for getting young children to think mathematically.*

Suggested Approach. Teachers can start a lesson with "What do you think?" or "How can we find an answer?" When children give an answer, teachers might ask, "How did you figure that out?" or "Show me how you did that." If children share a strategy, teachers might also ask, "Is there another way to solve that problem?" or "What would happen if I changed…?" Asking children to compare and contrast also helps them clarify their ideas ("How are these [shapes, numbers, patterns, measuring] tools alike or different?"). These questions are appropriate for any math content area.

Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

Dedicated time that is devoted to planned, daily math lessons can allow children to develop important skills in number and operations, geometry, patterns, measurement, and data analysis. By connecting math to a variety of everyday situations and routines, teachers can make math meaningful and provide opportunities for children to practice what they have learned in a purposeful manner.[131] If teachers coordinate their current math objectives with activities in the classroom and lessons in other subject areas, children can master skills and extend the concepts to higher levels or broader contexts.[132]

A classroom environment that contains math-related objects can help children recognize and apply math knowledge. For example, games can provide an enjoyable and meaningful way to learn a range of math ideas and practice a wide variety of basic skills.[133] Games can build on children's math knowledge, provide a reason for learning skills and concepts, supply repeated practice that is not boring, give children and teachers an opportunity to discuss strategies and ideas, and generate excitement.[134]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 18 randomized controlled trials[135] and 2 quasi-experimental studies[136] that met WWC standards and examined the effects of interventions that included dedicated time for math instruction, integration of math into other aspects of the school day, and use of games to practice math skills. Children in the studies attended preschool, prekindergarten, and kindergarten.

One of the studies examined *Math Is Everywhere*, a collection of 85 suggested activities (e.g., books, music, games, discussions, and group projects) that reinforce math concepts.[137] These activities can be implemented during various times of the day, such as circle time, transitions, or mealtimes. Children in classrooms using *Math Is Everywhere* scored higher in the general numeracy domain than children in classrooms where the teachers continued their regular classroom instruction. These higher scores could be due to teachers providing daily math lessons and incorporating math into various times of the day; however, the scores could also be due to aspects of other recommendations present in the intervention.

Another group of studies found that children who played number-based board games performed better in the domain of basic number concepts than did children who

played color-based board games or no board games.[138] However, the effects of number-based board games on measures of number recognition and operations were mixed.[139] The interventions in which playing a board game was part of a larger curriculum included not only elements of this recommendation but also other recommendations in the guide.[140]

The panel concluded that the body of evidence assessed in relation to Recommendation 5 was promising. However, the panel identified two limitations to the body of evidence. First, the examined interventions were multi-component interventions incorporating elements of other recommendations in the guide.[141] Second, in some studies there was a lack of clarity regarding the instruction the intervention and comparison groups received.[142] Despite these limitations, the panel recommends dedicating time to teach math, integrating math into other aspects of the day, and using games to practice math skills based on its expertise and the pattern of positive effects.

The panel identified five suggestions for how to carry out this recommendation.

How to carry out this recommendation

1. Plan daily instruction targeting specific math concepts and skills.

In order for preschool, prekindergarten, and kindergarten children to develop math skills, teachers should set aside time each day for purposeful math instruction.[143] Dedicated time for math instruction can help to provide children with skills in the foundational areas of math described in Recommendations 1 and 2. During math lessons, teachers can help children learn specific skills they can build upon throughout the rest of the day (as described in the remainder of this recommendation). Teachers can use large and small groups during dedicated math time to tailor instruction for children at different developmental levels.

Large-group (or whole-class) time can be a good place to introduce a concept for the first time or illustrate a concept through an example that is relevant to children's everyday lives. For example, teachers can read children a book that relates to the skills that will be taught, or they can play a whole-group game with the class. It is important to remember, however, that introducing a concept in a large group is most helpful when children have similar skill levels; it is also useful to reinforce the concept in smaller groups, particularly for children whose math understanding may not be as advanced as other children and who may miss key instructional points during whole-group activities.

After a particular concept is introduced in a large group, teachers should provide time for at least one small-group activity to help children practice and reinforce their skills. It may be particularly useful to broadly introduce a math concept during a large-group time, then tailor instruction to small groups of children who are at similar developmental levels so they can work on particular aspects of that skill, as described in Example 9.

2. Embed math in classroom routines and activities.

A daily or weekly schedule provides many opportunities to reinforce math concepts outside of the dedicated math instruction period.[144] Routines such as taking attendance can serve a math purpose in addition to a practical one. For example, teachers can engage children in using tally marks, beads, abacuses, or other markers to count how many girls, boys, and total children are in the classroom. After the count is decided, the teacher can extend math thinking

Example 9. Linking large groups to small groups

Objective

Understand the differences and similarities between triangles, rectangles, and squares.

Materials needed:

- Book: *Bear in a Square*, by Stella Blackstone
- A variety of other objects (based on availability, but could include the following)
  - Large pieces of paper cut into varied shapes for painting
  - Lunch trays and a small amount of sand
  - Geoboards with rubber bands

Directions, large group: Read the book in a large group, highlighting the names of all the shapes but focusing specifically on the difference between the number and length of sides and types of angles in triangles, rectangles, and squares.

Directions, small group: Once children are divided into small groups, highlight the number and length of sides and types of angles in each of the shapes the children create in the activities below. Children should be encouraged to use informal terms to describe the shapes, such as "long" and "short" sides and "big" and "little" angles for triangles. These activities will vary based on the types of materials available, but they could include the following:

- Provide paint, chalk, or other art materials so that children can add a stripe around the edge of a large paper cutout of a triangle or rectangle. Then, have the children continue to add more of the same shapes inside the original shape to create a design with concentric shapes.
- Lead children to use their fingers to draw shapes in sand on a tray or in a sandbox. They might draw shapes within shapes or combine shapes to make other figures.
- Encourage children to experiment with placing rubber bands on a geoboard to make triangles, rectangles, and squares of different sizes and orientations.

Early math content areas covered

- Geometry (shapes and attributes of shapes)

Monitoring children's progress and tailoring the activity appropriately

- For children who are more advanced, more complex shapes can be used. More advanced children may notice the number of sides on other shapes, such as a pentagon, or may ask about the number of sides in a circle.

Integrating the activity into other parts of the day

- Take a group walk outside to collect sticks of different sizes, and then use them to make and identify shapes.

Using the activity to increase math talk in the classroom

- Encourage the children to look around their environment, such as on tables in the classroom or on their clothing, to identify examples of triangles, rectangles, and squares. When children locate a shape, ask them to explain it to the group: "How can you tell that shape is a ___?" Prompt the children to identify the number and length of sides and type of angles.

by saying, for example, "We have 8 girls and we have 10 boys. We have 18 children all together: 8 plus 10 equals 18." The class could then display the results of attendance for several days using a chart that has columns or rows titled with the days of the week or a pie chart with the number of slices in the pie matching the total number of children in the class on a particular day. Teachers can also engage children in other everyday activities that may have a math component. For example, teachers can have children answer a yes/no "question of the day" every day. Children can then record how many of their classmates said "yes" and how many said "no" in a graph and compare the two numbers. Example 10 describes an opportunity to reinforce math concepts during snack time, another routine activity.

Example 10. Snack time

Objective

Practice counting, cardinality, addition, and subtraction.

Materials needed:

- Snacks
- Plates or paper towels

Directions: Once children receive an equal number of snacks, have them count how many they have. As they eat their snacks, they can compare how many they have relative to other children. Teachers can tailor snack time activities for use with the entire class or small groups.

Early math content areas covered

- Counting using one-to-one correspondence
- Cardinality
- Adding and subtracting (one more/fewer)

Monitoring children's progress and tailoring the activity appropriately

- Observe and note how each child counts the snacks. For example, does the child line up the pieces of the snack, or can the child count the pieces while they are scattered?
- Adapt this activity for children of varying levels by reducing the number of snack pieces to count or by asking each child different questions, such as "How many will you have after you eat one?" or "How many will you have after your friend gives you one?"

Integrating the activity into other parts of the day

- Ask children, "How many?" and "How can we find out how many?" whenever the opportunity arises. For example, ask, "How many books did you read?" or "How many children built this beautiful tower?"

Using the activity to increase math talk in the classroom

- Ask children to count out loud and compare amounts throughout the day to increase math talk in the classroom.

3. Highlight math within topics of study across the curriculum.

Teachers can integrate math concepts into non-math lessons by highlighting the aspects of math that are already present in the curriculum.[145] Teachers can point out opportunities to count objects, examine shapes, analyze data, or measure objects (depending on the current math objectives and where children are in the developmental progressions for these content areas).

During literacy time, for example, when reading a story, the teacher can ask questions that encourage children to solve a math problem based on the story. If the class is reading a story about the three little pigs, the teacher can ask the children to count the pigs, or the teacher could ask how many cupcakes they would need for a party with the three pigs. Teachers should select books that reinforce current math objectives. Teachers can also consider using more than one book to illustrate a given math concept, so children understand that a concept or skill can be applied in multiple contexts. Table 9 provides examples of ways to integrate different math content areas into literacy, science, art, health and safety, and social studies lessons.

Table 9. Integrating math across the curriculum

	Math Content Area				
	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Literacy	*We All Went on Safari*, Krebs *Mouse Count*, Walsh *7 Little Rabbits*, Becker and Cooney	*Bear in a Square*, Blackstone *Mouse Shapes*, Walsh *Shapes*, Silverstein	*A Pair of Socks*, Murphy *Pattern Bugs*, Harris *Pattern Fish*, Harris	*How Big Is a Foot?*, Myller *Spence Is Small*, Chevalier *Tall*, Alborough *The Grouchy Ladybug*, Carle	*It's Probably Penny*, Leedy *The Great Graph Contest*, Leedy *Tiger Math*, Nagda and Bickel
Science	Count collections of natural objects. Count how many days it takes for a plant sprout.	Describe objects from nature (e.g., rocks, leaves, and insects) in geometric terms. Use precut shapes to make animals.	Find and identify patterns in nature (e.g., on butterflies and snakes). Design a model of an insect using a pattern design.	Measure the growth of a plant in the classroom each day and predict how much time it will take before flowers will be visible on the plant.	Graph the amount the classroom plant grows each day. Graph animals with two legs, four legs, and more than four legs.
Art	Count how many objects appear in a piece of artwork.	Identify shapes in artwork. Decorate drawings of shapes.	Use patterns to make pictures or frames for pictures. Find and identify patterns in artwork.	Use measurement to make frames for art out of poster board or card stock.	Make a graph of the children's favorite colors. Tally children's opinions about artwork. For example, ask, "Which painting do you like better?"

(continued)

Table 9. Integrating math across the curriculum *(continued)*

	Math Content Area				
	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Health and Safety	Count the length of time it takes to wash your hands. List rules for washing hands or playing safely outside.	Use traffic signs to recognize shapes. Walk lines that are different shapes to practice balance control.	Jump rope or play hopscotch with an alternating pattern.	Measure your body's growth over time.	Graph your height or foot size.
Social Studies	In a unit about families, order people by size or from youngest to oldest. During a unit on recycling, children can count how many of a certain object they have collected to recycle.	Identify squares, straight lines, curved lines, etc., on maps.	Study patterns in clothes from different parts of the world. Look for patterns in flags from other countries.	Make a map of the neighborhood using measuring, geometry, spatial thinking, and positioning words.	Graph the size of the children's families. Make a graph that shows how children come to school (by bus, by car, etc.).

4. Create a math-rich environment where children can recognize and meaningfully apply math.

Teachers can provide opportunities for children to see and use math concepts regularly by creating a math-rich classroom environment. This enrichment can be done by making math-related objects and tools readily available, labeling and organizing math-related objects and tools so they are easy to find and use, and organizing activities and routines with numeric systems.[146]

Teachers should provide a variety of tools throughout the classroom to allow children to explore each of the five math content areas. Table 10 lists examples of tools for different math content areas.

Table 10. Examples of tools that can be useful in each math content area

	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Objects and Tools	blocks abacuses number lists number puzzles	geoshapes precut foam shapes traffic signs for classroom areas	beads different-colored cubes art materials, such as stamps and markers	rulers tape measures clocks scales measuring spoons and cups	clipboard and paper for tallying the "question of the day" hula hoops or small hoops that bend for Venn diagrams sorting bins

Teachers can explicitly teach children how to use tools by modeling their use during small- or large-group time.[147] For example, the teacher can use shapes or blocks to demonstrate how a rectangle and a triangle can be combined to make a house. As another example, the teacher can bring different types of measuring tools to circle time to demonstrate how to use tools to measure objects of varying sizes (e.g., placing the ruler next to the object to be measured, with the end of the ruler at one end of the object, then reading the number closest to the opposite edge of the object).

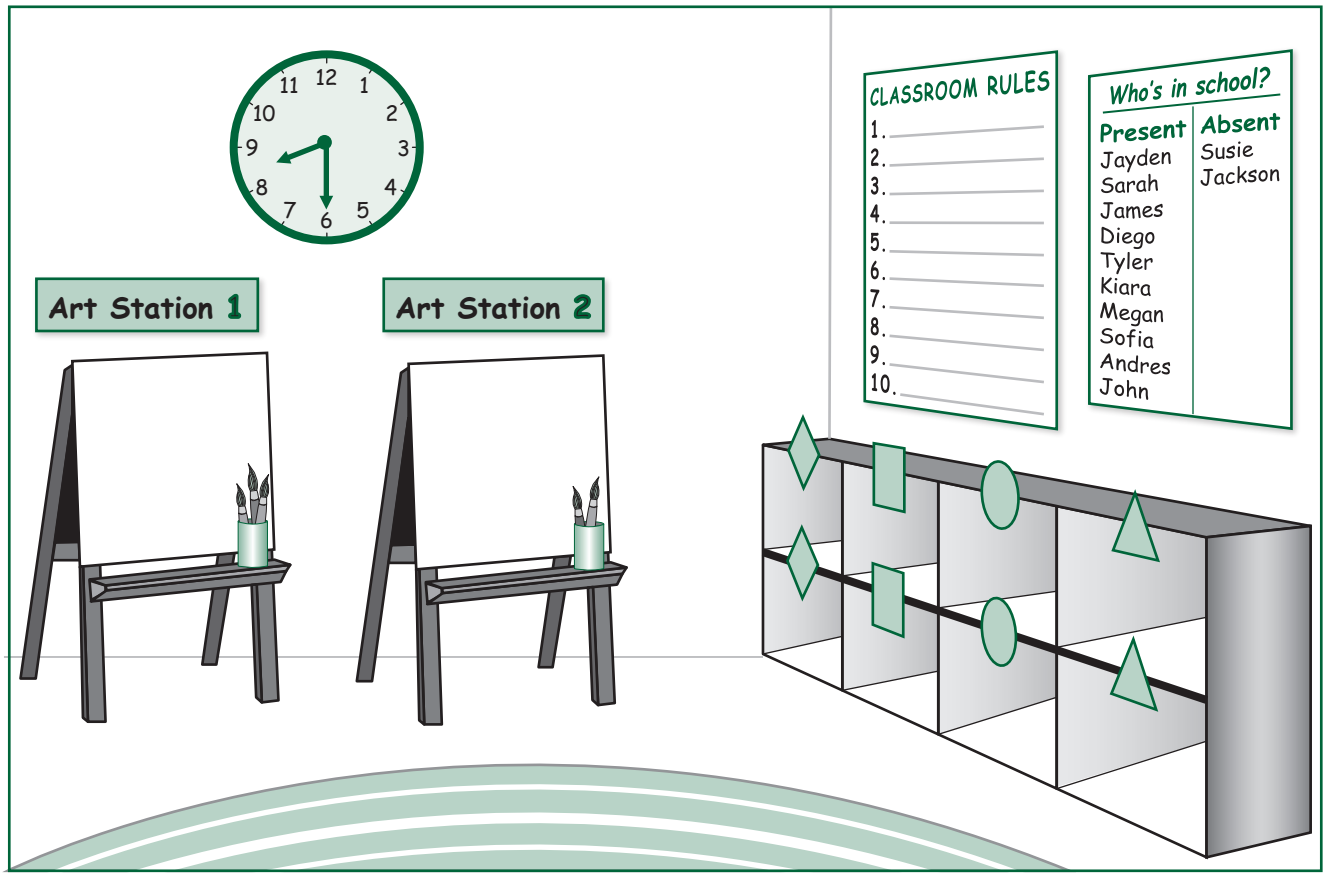
Teachers can place tools, such as number lists, rulers, and scales, at eye level for children. Also, the classroom can be organized and labeled in a manner that supports learning. For example, lunch tables can be labeled with shapes, and children can sit at the "triangle table" or the "circle table" for lunch.

After children learn those shapes, the labels can be changed to new shapes. These activities, along with activities described in Recommendation 4, help children learn and apply math vocabulary in meaningful ways.

Organizing activities and routines with numeric systems can give children opportunities to reinforce and practice math concepts while becoming more independent. To do this, teachers can display charts with sequenced directions and picture icons, number the classroom rules on a poster, or use a numerical system to indicate how many children can work in a center at the same time, as displayed in Figure 7.

Teachers can also involve children in labeling and organizing the environment as much as possible. For example, teachers can discuss how many people can safely work in a particular center, then have the children help make the label and the number for it.

Figure 7. An example of a math-rich environment in the classroom



5. Use games to teach math concepts and skills and to give children practice in applying them.

Games can provide an engaging opportunity to practice and extend skills. If children have fun playing the games, they are more likely to be motivated to practice math.[148] For maximum benefit, teachers should select specific games to match current math objectives. Example 11 provides an example of a game (*Animal Spots*) that reinforces one-to-one correspondence and cardinality. Games that target different math content areas are often included in math curricula. Games can also be purchased separately or be made by the teacher. Some math concepts may also be highlighted in games that come up during natural play, such as hopscotch or jump rope.

Example 11. The *Animal Spots* game[149]

Objective

Practice one-to-one correspondence and cardinality

Materials needed:

- Pictures of animals or materials children can use to draw their own animals
- Small circles of paper to use as spots
- Glue
- A die or spinner to determine the number of spots to place on each animal

Directions: Have each child draw the outline of an animal on a piece of paper, or provide handouts with large outlines of animals. Each child should take a turn throwing the die to determine how many spots to place on his or her animal. The children should count out the number of dots on the face of the die, and then they should choose the same number of "spots" from a bowl of paper circles in the center of the table. After children have selected the correct number of spots, they can glue them onto their animals. Teachers can tailor the *Animal Spots* game for use with the entire class, a small group, or individual children.

Early math content areas covered

- Counting using one-to-one correspondence
- Cardinality

Monitoring children's progress and tailoring the activity appropriately

- Observe the play, noting each child's ability to count the number of dots on the die and count out the same number of spots from a larger pile.
- Use one die or a spinner at the beginning; then, use two dice to increase difficulty.

Integrating the activity into other parts of the day

- Have children count out objects from a larger set. For example, a child can choose ten blocks for building or five shapes from a larger collection to use for a collage.

Teachers can get involved with the game-playing to ensure educational play. For example, if children are playing a game to learn one-to-one correspondence and cardinality, the teacher can emphasize moving one space at a time and then reinforce the total number of spaces that the game piece should be moved. The teacher can also use the game to extend children's skills. For example, if children are ready, the teacher can use a pair of dice instead of a single die or a spinner, so children have to count and add the dots on each die.

Potential roadblocks and solutions

Roadblock 5.1. *The school is on a limited budget and cannot afford to purchase many classroom materials or games.*

Suggested Approach. Math can be embedded in the classroom without spending a lot of money if teachers take advantage of opportunities that occur naturally throughout the day. For example, teachers can highlight math concepts that come up in an already-planned literacy or science lesson by asking children a question that requires them to use math concepts. In addition, teachers may be able to create games on their own with readily available natural materials such as leaves, sticks, and rocks.

When purchasing materials, strategic planning can help save resources. Teachers can choose games that teach the math content areas children are most interested in. They can also choose games that are accessible to a range of skill levels to avoid having to purchase more than one game. For example, if the teacher is playing a memory game with younger or less advanced children, the group can play with all the cards face-up, or they can play with fewer cards than the whole set. The teacher can play the same game with older or more advanced children by flipping the cards over and using the whole set.

Teachers can also turn to existing community resources. For example, they can take advantage of the local public library to find math-related books for their classroom. Many librarians can help teachers by selecting books related to certain topics requested by the teacher. Also, some communities may also have toy-lending libraries from which teachers may borrow games or other manipulatives.

Roadblock 5.2. *I am told that it is important to provide literacy-, science-, art-, and math-rich environments. It is difficult to keep all subjects in mind at all times.*

Suggested Approach. Teachers do not need to include all aspects of all subjects at one time. Instead, they can rotate the activities and materials in the classroom based on the instructional objectives at that particular time. They can also try to coordinate the use of materials and activities to meet multiple goals. For example, reading a story that contains math content areas can help meet a math objective and a literacy objective simultaneously. When lesson planning, teachers can select ahead of time the learning objectives they would like to focus on each day and then plan activities and modify the classroom environment to support those objectives.

Roadblock 5.3. *I do not have my own space because multiple classes use the same classrooms throughout the day.*

Suggested Approach. If the classroom environment cannot be modified, teachers should take advantage of ways to embed math concepts that do not involve modifying the classroom environment. Alternatively, teachers could use a mobile chart stand to hold multiple charts that could be displayed throughout the day.

Roadblock 5.4. *Parents may wonder why their children are playing games in school.*

Suggested Approach. Teachers should help parents understand the importance of play in motivating children to practice concepts they are learning in more formal math instruction. Teachers can help alleviate parental concerns by selecting games with certain objectives in mind, so when a parent asks why a certain game is being played at school, teachers can respond accordingly. For example, a teacher might say, "We are playing *Go Fish* because it helps the children recognize numbers, match numbers, and determine, at the end of the game, who has more matches and who has fewer matches." Teachers can also use board games to support children in learning numbers and counting. For an example of a game that a teacher could make, see Siegler and Ramani (2009).

Glossary

A

An **assessment** provides information on how much a child knows about a particular topic or the skills a child has in a particular area. Assessments may include an adult's observation of a child in classroom activities, an adult's rating of the child, or an adult's scoring of a child's accuracy on a particular task (e.g., test or worksheet). Assessments may be *formal*, such as standardized tests, standardized rating scales, teacher-developed tests, or worksheets. Teachers may also conduct *informal* assessments to check to see what a child knows or can do. Assessments can be *formative*, with the results used to determine the extent to which the child learned the intended skills from instruction as part of progress monitoring. Finally, assessments may be *summative*, with the result documenting a child's performance, for example, on an end-of-chapter test or state developed test. The particular type of assessment (formal or informal, formative or summative) should be chosen based on how the results will be utilized.

C

Cardinality is the total number of items in a collection. The **cardinality principle** is the understanding that when counting, the number word assigned to the last item of a collection represents the total quantity.

A **collection** is a group of discrete objects or things.

D

A **developmental progression** refers to a sequence of skills and concepts that children acquire as they build math knowledge. It effectively defines the developmental prerequisites for a skill or concept.

For grouping outcomes within WWC reviews for this practice guide, the panel defines a **domain** as a group of outcomes related to a child's math achievement. For this practice guide, the panel has identified six domains into which all outcomes are grouped: general numeracy, basic number concepts, number recognition, operations, geometry, and patterns and classification. The domains are not meant to be synonymous with any early math content area (see **early math content areas**).

E

Early math content areas are the specific math topics the panel believes should become the foundation of preschool, prekindergarten, and kindergarten curricula. The panel has identified number and operations, geometry, patterns, measurement, and data analysis as critical to children's math learning. Outcome domains defined for grouping outcomes in WWC reviews cover the range of skills within the early math content areas, but in some cases, the skills are grouped slightly differently (see **domain**).

F

Formal representations are the typically school-taught standard mathematical terms and symbols that represent mathematical ideas. **Informal representations** are familiar everyday objects, pictures, or words that stand for those ideas. **Informal units**, a type of informal representation, are non-standard forms of measurement, such as blocks or children's hands and feet. By contrast, examples of formal or standard measurement tools include rulers and scales. **Informal methods** are children's self-invented strategies to solve mathematical problems, and these may be supported and encouraged by teachers.

I

The **increasing magnitude principle** is the idea that a number word later in the counting sequence represents a larger quantity than a number word earlier in the counting sequence.

M

Math knowledge is a child's understanding of math concepts and skills. **Math achievement** refers to a child's performance on a variety of math tasks, including assessments.

A **multi-component intervention** is a set of instructional practices that are implemented together and evaluated as a set.

N

A **non-example** illustrates what a concept is not. For example, whereas five and six come after four and are examples of numbers larger than four, two and three come before four and are not larger. Non-examples are teaching tools designed to illustrate the difference between two things, and thus to help children learn the boundaries of a concept.

Number refers to a system for representing quantity. **Number knowledge** consists of an understanding of numbers and the relations among them. It includes the ability to recognize quantity, count, identify numerals (written numbers), and perform number operations.

Number-after knowledge is a counting skill that comes from experience with the number sequence. Children with number-after knowledge are able to identify the next number in the counting sequence without starting the count from one.

A **number list** is a series of numerals beginning with 1 and ordered by magnitude.

Number sense refers to a person's general understanding of number and operations along with the ability to use this understanding in flexible ways to make math judgments and to develop useful strategies for solving complex problems.[150]

Numerals, or written numbers, are symbols that represent numbers. For example, the numeral 8 is the symbol that represents the number eight.

O

The **one-to-one counting principle** refers to understanding one-to-one correspondence; that is, when counting, each item in a collection must be assigned one and only one number word.

The panel uses the term **operations** to refer to arithmetic. Addition and subtraction are examples of operations.

P

Prekindergarten (Pre-K) refers to the year before children enter kindergarten, usually when children are 4 years old. **Preschool** refers to the year before the prekindergarten year, when most children are 3 years old.

Progress monitoring is a systematic approach to assessment with the goal of improving skills. Progress monitoring begins with an evaluation of the child's current level of knowledge. Changes in the child's skills are then tracked through regular assessment, and goals and teaching strategies are adjusted based on the child's progress.

S

Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. For example, subitizing enables a child to see a collection of three toy animals and immediately know, without counting, that there are three.[151] This ability is also known as small-number recognition.

Appendix A

Postscript from the Institute of Education Sciences

What is a practice guide?

The Institute of Education Sciences (IES) publishes practice guides to share evidence and expert guidance on addressing education-related challenges not readily solved with a single program, policy, or practice. Each practice guide's panel of experts develops recommendations for a coherent approach to a multifaceted problem. Each recommendation is explicitly connected to supporting evidence. Using common standards, the supporting evidence is rated to reflect how well the research demonstrates the effectiveness of the recommended practices. Strong evidence means positive findings are demonstrated in multiple well-designed, well-executed studies, leaving little or no doubt that the positive effects are caused by the recommended practice. Moderate evidence means well-designed studies show positive impacts, but there are questions about whether the findings can be generalized beyond the study samples or whether the studies definitively show evidence that the practice is effective. Minimal evidence means that there is not definitive evidence that the recommended practice is effective in improving the outcome of interest, although there may be data to suggest a correlation between the practice and the outcome of interest. (See Table 1 for more details on levels of evidence.)

How are practice guides developed?

To produce a practice guide, IES first selects a topic. Topic selection is informed by inquiries and requests to the What Works Clearinghouse Help Desk, formal surveys of practitioners, and a limited literature search of the topic's research base. Next, IES recruits a panel chair who has a national reputation and expertise in the topic. The chair, working with IES, then selects panelists to co-author the guide. Panelists are selected based on their expertise in the topic area and the belief that they can work together to develop relevant, evidence-based recommendations. IES recommends that the panel include at least one practitioner with expertise in the topic.

The panel receives a general template for developing a practice guide, as well as examples of published practice guides. Panelists identify the most important research with respect to their recommendations and augment this literature with a systematic search for studies assessing the effectiveness of particular programs or practices. These studies are then reviewed against the What Works Clearinghouse (WWC) standards by certified reviewers who rate each effectiveness study. WWC staff assist the panelists in compiling and summarizing the research and in producing the practice guide.

IES practice guides are then subjected to external peer review. This review is done independently of the IES staff that supported the development of the guide. A critical task of the peer reviewers of a practice guide is to determine whether the evidence cited in support of particular recommendations is up-to-date and that studies of similar or better quality that point in a different direction have not been overlooked. Peer reviewers also evaluate whether the level of evidence category assigned to each recommendation is appropriate. After the review, a practice guide is revised to meet any concerns of the reviewers and to gain the approval of the standards and review staff at IES.

A final note about IES practice guides

In policy and other arenas, expert panels typically try to build a consensus, forging statements that all its members endorse. Practice guides do more than find common ground; they create a list of actionable recommendations. Where research clearly shows which practices are effective, the panelists use this evidence to guide their recommendations. However, in some cases research does not provide a clear indication of what works. In these cases, the panelists' interpretation of the existing (but incomplete) evidence plays an important role in guiding

the recommendations. As a result, it is possible that two teams of recognized experts working independently to produce a practice guide on the same topic would come to very different conclusions. Those who use the guides should recognize that the recommendations represent, in effect, the advice of consultants. However, the advice might be better than what a school or district could obtain on its own. Practice guide authors are nationally-recognized experts who collectively endorse the recommendations, justify their choices with supporting evidence, and face rigorous independent peer review of their conclusions. Schools and districts would likely not find such a comprehensive approach when seeking the advice of individual consultants.

Institute of Education Sciences

Appendix B

About the Authors

Panel

Douglas Frye, Ph.D., is an associate professor at the University of Pennsylvania's Graduate School of Education and is the director of the Interdisciplinary Studies in Human Development program. Dr. Frye's research efforts are concentrated on two topics in cognitive development: children's early math development and theories of mind. His math research focuses on the developmental sequence of early math reasoning skills and activities to support young children's development of those skills. Dr. Frye has been involved in designing and evaluating several strategies-based emergent numeracy interventions, including a computer-based program (*Kids Count I* at Yale University) as well as classroom-based interventions implemented in urban Head Start classrooms (e.g., *Kids Count II* and the *Evidence-based Program for Integrated Curricula [EPIC]* at the University of Pennsylvania). Dr. Frye also investigates how young children's theories of mind relate to their understanding of teaching and learning. He has been an associate editor of *Child Development* and the *Journal of Cognition and Development*.

Arthur J. Baroody, Ph.D., is Professor Emeritus of Curriculum and Instruction at the University of Illinois at Urbana-Champaign and a Senior Research Fellow in the Morgridge College of Education at the University of Denver. His research focuses on the teaching and learning of basic counting, number, and arithmetic concepts and skills by young children and children with learning disabilities. Dr. Baroody is currently the co-principal investigator of a post-doctoral grant, both funded by the U.S. Department of Education. He has recently served as the principal investigator for grants from the Institute of Education Sciences, the National Institutes of Health, the National Science Foundation, and the Spencer Foundation. Additionally, Dr. Baroody is the author of several books on teaching math to children and the co-author of the Test of Early Mathematics Ability, third edition. He also co-edited a book on mathematics learning that is part of the *Studies in Mathematics Thinking and Learning* series. Articles that Dr. Baroody has authored or co-authored have recently been published in the *American Educational Research Journal*, *Cognition and Instruction*, *Developmental Disabilities Research Reviews*, *Journal of Educational Psychology*, and *Mathematics Thinking and Learning*.

Margaret Burchinal, Ph.D., is a senior scientist and the director of the Design and Statistical Computing Unit at the Frank Porter Graham Child Development Institute, as well as a research professor in the psychology department at the University of North Carolina. She was a professor in the department of education at the University of California, Irvine, from 2007 to 2011. As an applied methodologist, Dr. Burchinal helped demonstrate that sophisticated methods such as meta-analysis, fixed-effect modeling, hierarchical linear modeling, piecewise regression, and generalized estimating equations provide educational researchers with advanced techniques to address important educational issues, such as whether child care quality measures are biased. She served as the primary statistician for many educational studies of early childhood, including the Prekindergarten Evaluation for the National Center for Early Learning and Development; the Cost, Quality, and Child Outcomes Study; the Family Child Care and Relative Care Study; and the Abecedarian and CARE Projects. Dr. Burchinal served as a member of the National Academy of Science's Committee on Developmental Outcomes and Assessments for Young Children. Boards she has served on or is currently serving on include the advisory board for the National Center for Educational Statistics, the advisory council for Head Start Research, the advisory board for the Research Bureau of Maternal and Child Health, the technical work group for Early Reading First Evaluation, and the advisory board for the Los Angeles Universal Prekindergarten Program. She also currently serves on the editorial board for *Child Development* and the *Early Childhood Research Quarterly*.

Sharon M. Carver, Ph.D., is the director of the Carnegie Mellon University Children's School. As director, she is currently combining precise goal specification, instructional design, and focused assessment to explore how young children's development of problem-solving skills can be enhanced to promote general transfer. Dr. Carver and the teachers have designed their three-year early childhood curriculum and assessment framework to focus more directly on the cognitive processes and rich knowledge base that provide an essential foundation for academic success after kindergarten. Her research bridges the domains of cognitive development and educational psychology, focusing on using models of cognitive skills to design instruction and assessments that will facilitate skill acquisition and transfer in school contexts. In prior research, Dr. Carver collaborated with teachers in an urban school to design an innovative curriculum and learning environment for middle school. She consults widely with educators developing diverse programs for learners of all ages.

Nancy C. Jordan, Ed.D., is a professor of education at the University of Delaware. She is the principal investigator of the Number Sense Intervention Project, funded by the National Institute of Child Health and Human Development, as well as the Center for Improving Learning of Fractions, funded by the Institute of Educational Sciences. Dr. Jordan is the author or co-author of many articles in children's math and has recently published articles in *Child Development*, the *Journal of Learning Disabilities*, *Developmental Science*, *Developmental Psychology*, and *School Psychology Review*. Recently, Dr. Jordan served on the committee on early childhood math for the National Research Council of the National Academies.

Judy McDowell, M.E., has been a Philadelphia Head Start teacher for 18 years and worked with parents and children as an early interventionist prior to joining Head Start. Ms. McDowell was teacher of the year in her region of Philadelphia (University City) in 2002, was nominated as teacher of the year for all of Philadelphia Head Start in 2002, and was twice nominated as teacher of the year for the School District of Philadelphia (2001 and 2004). Throughout her career, Ms. McDowell has welcomed research teams into her classroom and developed collaborative relationships with researchers. As a consultant with the *Evidence-based Program for Integrated Curricula (EPIC)* project, she translated research-based developmental principles into early childhood classroom–friendly math, literacy, and socio-emotionally focused activities. She has mentored other teachers implementing this curriculum and leads professional development meetings. Ms. McDowell is also a COR assessment trainer/mentor for Philadelphia Head Start. She has co-authored a book chapter with Ageliki Nicoloupolo and Carolyn Brockmeyer on how play motivates and enhances children's development—based on research conducted in her classroom. She has presented at professional conferences including National Head Start Association conferences and the International Conference on Imagination and Education. Ms. McDowell is certified in early childhood education, elementary education, and special education.

Staff

M. C. ("Cay") Bradley, Ph.D., is a researcher at Mathematica Policy Research. She has both delivered and evaluated education and social work programs. Dr. Bradley supported the panel in the review and documentation of evidence. She has reviewed evidence for previous What Works Clearinghouse practice guides and topic areas. Dr. Bradley has also conducted or participated in other meta-analyses and syntheses focused on paraprofessional home-visiting programs and interventions for oppositional defiant disorder.

Elizabeth W. Cavadel, Ph.D., is a researcher at Mathematica Policy Research. She has a background in child development and psychology. Dr. Cavadel assisted the panel in the writing of this practice guide, drawing on her experience in early numeracy and in translating research to practice. Dr. Cavadel has worked on large-scale

psychological and educational interventions and evaluations in Head Start and private preschool settings. Her current work focuses on child outcomes across a range of topics including early child-care quality, children's school readiness, child behavior, social-emotional development, and early childhood assessment. Dr. Cavadel is a certified What Works Clearinghouse reviewer and has also reviewed evidence and synthesized reports focusing on home-visiting programs.

Julia Lyskawa, M.P.P., is a research analyst at Mathematica Policy Research. She assisted in the research and writing of this practice guide. Ms. Lyskawa currently works on three other education-focused projects, including an evaluation of child outcomes and coaching methods in the Los Angeles Universal Preschool Program, a feasibility study of extending learning time in elementary schools, and a redesign of the Head Start Family and Child Experiences Survey (FACES).

Libby Makowsky, M.P.P., is a researcher at Mathematica Policy Research. She has experience providing research support for various projects on topics related to out-of-school-time programs, teacher residency programs (TRP), and alternative teacher certification programs such as Teach for America (TFA). Drawing on experiences gained as a former classroom teacher, Ms. Makowsky assisted the panel in the writing of this practice guide. She is also a certified What Works Clearinghouse reviewer.

Moira McCullough, M.P.P., is a research analyst at Mathematica Policy Research. She assisted the panel in the writing of the technical appendices for this practice guide. She is a certified What Works Clearinghouse reviewer and is a deputy principal investigator for the Early Childhood Education topic area. She has experience assisting with evidence documentation and writing for numerous practice guides on topics such as effective fractions instruction.

Marc Moss, Ed.D., is a researcher at Abt Associates. Dr. Moss has directed numerous large-scale, national evaluations that examined the implementation and impact of various reforms in the field of education. He has reviewed evidence for and participated in the writing of this practice guide. Dr. Moss has also reviewed evidence for other What Works Clearinghouse practice guides and topic areas.

Bryce Onaran, M.P.A., is a survey specialist at Mathematica Policy Research. He has served as staffing coordinator for the What Works Clearinghouse, where he managed the planning and operation of the project. Mr. Onaran led the panel's efforts to translate research findings into practitioner-friendly text. In addition to his work on the What Works Clearinghouse, Mr. Onaran also works on data collection and program evaluation efforts, primarily in the area of education.

Michael Barna, M.A., is a research analyst at Mathematica Policy Research. He assisted the panel in gathering evidence and providing logistical support. Previously, Mr. Barna has served as coordinator for the elementary and middle school math topic areas. He also has experience providing research support and conducting data analysis for studies of charter management organizations.

Appendix C

Disclosure of Potential Conflicts of Interest

Practice guide panels are composed of individuals who are nationally recognized experts on the topics about which they are making recommendations. IES expects the experts to be involved professionally in a variety of matters that relate to their work as a panel. Panel members are asked to disclose these professional activities and institute deliberative processes that encourage critical examination of their views as they relate to the content of the practice guide. The potential influence of the panel members' professional activities is further muted by the requirement that they ground their recommendations in evidence that is documented in the practice guide. In addition, before all practice guides are published, they undergo an independent external peer review focusing on whether the evidence related to the recommendations in the guide has been presented appropriately.

The professional activities reported by each panel member that appear to be most closely associated with the panel recommendations are noted below.

Arthur J. Baroody served on the advisory panel during the development phase of *Building Blocks*, a curriculum reviewed in this guide. Dr. Baroody does not receive royalties or any other financial considerations from the sale of *Building Blocks*. Dr. Baroody is also the co-author of the Test of Early Mathematics Ability—Third Edition (TEMA-3), which is used as an outcome in studies reviewed in this guide. He receives royalties on the sale of TEMA-3 from PRO-ED, Inc.

Douglas Frye and **Elizabeth Cavadel** collaborated on developing the initial mathematics portion of *Evidence-based Program for Integrated Curricula (EPIC)*. **Judy McDowell** was involved in the development of *EPIC* as a classroom teacher. As *EPIC* is not currently available for purchase, none of these authors receive royalties for the curriculum.

Nancy Jordan was involved in the development of a number sense curriculum that is reviewed in the guide. As the curriculum is not currently available for purchase, Dr. Jordan does not receive royalties for the curriculum. Dr. Jordan is also the co-developer of the *Number Sense Screener* assessment tool. She receives royalties on the sale of *Number Sense Screener* from Brookes Publishing.

Appendix D

Rationale for Evidence Ratings[152]

This appendix discusses studies that examined the effectiveness of recommended practices using strong designs for addressing questions of causal inference including randomized controlled trials (RCTs) and quasi-experimental designs (QEDs) that met What Works Clearinghouse (WWC) standards and were used to determine the level of evidence rating. The studies were identified through an initial search for research on practices for improving young children's early math achievement. The search focused on studies published between 1989 and 2011 that examined practices for teaching number, operations, and other early math content areas to children in preschool, prekindergarten, and kindergarten. Studies examined children in both the United States and other countries. Interventions could target children who were typically developing, at risk of facing challenges in math, or exhibiting challenges with math or school in general. The search was supplemented with studies recommended by the panel based on its expertise in the area of early math.

The panel identified more than 2,300 studies through this search, including 78 studies with rigorous designs reviewed according to WWC standards. Twenty-eight of these studies met evidence standards with or without reservations and tested interventions related to one or more recommendations. Study effects were calculated and classified as having a positive or negative effect when the result was either:

- statistically significant[153] or
- substantively important as defined by the WWC.[154]

When a result met none of these criteria, it was classified as having "no discernible effects." A study was described as having "mixed effects" if it had any combination of positive, negative, and no discernible effects.

Some studies met WWC standards but did not adjust statistical significance when there were multiple comparisons or when the unit of assignment was different from the unit of analysis ("clustering," for example, when classrooms are assigned to conditions but individual children's test scores are analyzed without accounting for the clustering of children in classrooms). When full information was available, the WWC adjusted for multiple comparisons and clustering within a domain.[155]

Eligible outcomes. The panel was interested in interventions demonstrating improvements in any aspect of a child's early math achievement. The guide focuses on six outcome domains.[156] The outcome domains reflect specific math concepts (geometry, operations, patterns and classification) as well as general numeracy or general math. The six outcome domains for this practice guide are as follows:

- The *general numeracy* domain includes measures that assess a child's overall numeracy or math achievement. For example, overall scores on the Test of Early Mathematics Ability (TEMA[157]) would fall into this domain, although subscales may be placed in other domains.
- The *basic number concepts* domain includes measures that assess a child's ability to understand fundamental characteristics of numbers. The measures could focus on counting, magnitude, or number-line estimation.
- The *number recognition* domain includes measures that assess a child's ability to identify numbers in specific forms: as a set, visually as a numeral, or verbally.
- The *operations* domain includes measures that assess a child's ability to perform addition and subtraction mentally or with sets of objects.
- The *geometry* domain includes measures that assess a child's ability to identify shapes and understand shape attributes (e.g., that squares have four sides of equal length).

- The *patterns and classification* domain includes measures that assess a child's ability to identify, replicate, and extend patterns. Also included are assessments of a child's ability to sort—for example, placing all red blocks on one shelf or all triangle blocks on another shelf.

Many of the studies reviewed by the panel included multiple outcomes, used the same outcomes at multiple points, or reported on both total and subscale scores. To facilitate comparisons, the panel focused on the outcome closest to the end of the intervention; these are labeled *posttests*. All outcome measures administered after the posttest are labeled *maintenance* in appendix tables. The panel prioritized findings of total or full-scale scores in the appendix tables. If both a total score and subscale scores were reported, the subscale findings are described as notes in the appendix tables.

Multi-component interventions. Many of the studies that contributed to the evidence ratings for this guide examined the effectiveness of several instructional practices tested together. The effects associated with multi-component interventions that included more than one of the panel's recommendations are viewed by the panel as support for the idea that all recommendations should be implemented together. For example, 8 of 28 studies that contributed to the level of evidence ratings contributed to the level of evidence rating for all five recommendations.[158] An additional 5 of 28 studies contributed to the level of evidence rating for four of the five recommendations.[159] These 13 studies tested the effectiveness of number and operations instruction designed to follow a developmental progression (Recommendation 1).[160] This was combined with instruction in other early math content areas and was designed to follow specific developmental progressions for each specific early math content area (Recommendation 2). The interventions also included regular assessments of the children's understanding, and they supported teachers in tailoring instruction (Recommendation 3). The interventions taught children to view and describe their world mathematically, providing math vocabulary as well as opportunities to talk about math (Recommendation 4). In addition, the interventions emphasized dedicating time for math instruction as well as incorporating math throughout the school day (Recommendation 5).

In studies of multi-component interventions, the panel could not identify which of the practices included in the interventions caused the observed effects on math outcomes. However, when these interventions led to positive effects on math outcomes, they provided evidence that at least one of the practices was effective, although it was not possible to identify which practice or practices were responsible for the effects seen. Table D.1 presents a summary of recommendations for which each study contributed evidence. If a study contributed to more than one recommendation, then it examined the effects of a multi-component intervention. In cases where a particular intervention led to negative or no discernible effects, the panel considered these effects when weighing the strength of the evidence for a specific practice.

The panel reviewed 13 studies that provided evidence for the effectiveness of nine curricula, which are related to at least four of the five recommendations and thus are examples of multi-component interventions. Below, the panel briefly describes each curriculum. Specific aspects of the curricula related to particular recommendations are highlighted in subsequent discussions about the evidence for each recommendation.

- *Bright Beginnings* is an early childhood curriculum, based in part on *High/Scope* and *Creative Curriculum*, with an additional emphasis on literacy skills. The curriculum consists of nine thematic units designed to enhance children's cognitive, social, emotional, and physical development. Each unit includes concept maps, literacy lessons, center activities, and home activities. Special emphasis is placed

Table D.1. Summary of studies contributing to the body of evidence, by recommendation

Citation	Contributes to the body of evidence for				
	Rec. 1	Rec. 2	Rec. 3	Rec. 4	Rec. 5
Arnold et al. (2002)	X		X	X	X
Aunio, Hautamaki, and Van Luit (2005)	X				X
Barnett et al. (2008)	X	X		X	X
Baroody, Eiland, and Thompson (2009)	X				
Casey et al. (2008)		X			
Clements and Sarama (2007b)	X	X	X	X	X
Clements and Sarama (2008)	X	X	X	X	X
Clements et al. (2011)	X	X	X	X	X
Curtis, Okamoto, and Weckbacher (2009)	X				
Dyson, Jordan, and Glutting (2013)	X		X	X	X
Fantuzzo, Gadsden, and McDermott (2011)	X	X	X	X	X
Fuchs, L. S., Fuchs, D., and Karns (2001)				X	
Griffin, Case, and Capodilupo (1995) and related publication Griffin, Case, and Siegler (1994)	X		X	X	X
Jordan et al. (2012)	X		X	X	X
Kidd et al. (2008)	X	X			
Klein et al. (2008)	X	X	X	X	X
Lai, Baroody, and Johnson (2008)	X				
Monahan (2007)	X				X
PCER Consortium (2008, Chapter 2)	X	X	X	X	X
PCER Consortium (2008, Chapter 3)	X	X	X	X	X
Ramani and Siegler (2008)	X				X
Ramani and Siegler (2011)					X
Sarama et al. (2008)	X	X	X	X	X
Siegler (1995)				X	
Siegler and Ramani (2008)	X				X
Siegler and Ramani (2009)					X
Sood (2009)	X				
Sophian (2004)	X	X		X	X
Weaver (1991)		X			

X The comparison was included in the body of evidence for this recommendation.

on the development of early language and literacy skills, and parent involvement is a key component of the program.[161]

- *SRA Real Math Building Blocks PreK* (also referred to as *Building Blocks for Math*) is a supplemental math curriculum designed to develop preschool children's early math knowledge through various individual and small- and large-group activities. It uses *Building Blocks for Math PreK* software, manipulatives, and print material. *Building Blocks for Math* embeds math learning in children's daily activities, ranging from designated math activities to circle and story time, with the goal of helping children relate their informal math knowledge to more formal math concepts.[162]
- The *Creative Curriculum for Preschool* is a project-based early childhood curriculum designed to foster the development of the whole child through teacher-led, small- and large-group activities. The curriculum

provides information on child development, working with families, and organizing the classroom around 11 interest areas, including math. Child assessments are an ongoing part of the curriculum, and an online program provides record-keeping tools to assist teachers with the maintenance and organization of child portfolios, individualized planning, and report production.[163]

- The researcher-developed, measurement-focused *Curriculum for Head Start* involves the use of alternative units and applies the concept of unit to enumeration, measurement, and the relationships between geometric shapes. Children are introduced to charts and graphs in the later part of the year to record observations and identify relationships. Two key concepts in the curriculum are (1) that the choice of unit will affect the numerical result from counting or other operations, and (2) that units can be combined to form higher-order units or taken apart to form lower-order units. Teachers are provided with weekly project activities, including supplemental activities and home activities.[164]
- The *Evidence-based Program for Integrated Curricula (EPIC)* is a stand-alone preschool curriculum developed for Head Start children that is designed to improve their math, language, and literacy. It is a unified program that intends to systematically incorporate content, instruction, professional development, and regular criterion-referenced assessments.[165]
- *Math Is Everywhere* is a curriculum that strives to incorporate math in the normal classroom routine. Teachers select from 85 activities that are designed to be fun for the children and developed to use different approaches to teach math, including books, music, games, and discussion. The activities cover early math concepts such as counting, recognizing and writing numbers, one-to-one correspondence, comparison, and change operations.[166]
- *Pre-K Mathematics* is a supplemental curriculum designed to develop informal math knowledge and skills in preschool children.[167] Math content is organized into seven units. Specific math concepts and skills from each unit are taught in the classroom through teacher-guided, small-group activities with concrete manipulatives. Take-home activities with materials that parallel the small-group classroom activities are designed to help parents support their children's math development at home.[168]
- *Rightstart* is a kindergarten curriculum composed of 30 interactive games that children can play to support learning about addition and subtraction, either as a whole class or in small groups with teacher supervision. Central instructional principles include bridging a child's current knowledge and the targeted information; use of props to support children's learning in a diverse manner; different levels to support variation in children's knowledge; cognitive and affective engagement of the children while learning; opportunities for children to interact with the props and use their knowledge constructively through physical, social, and verbal interactions; and the use of a developmental progression (the central conceptual structure) to sequence activities.[169]
- *Tools of the Mind* is an early childhood curriculum for preschool and kindergarten children, based on the ideas of Russian psychologist Lev Vygotsky. The curriculum is designed to foster children's executive functioning, which involves developing self-regulation, working memory, and cognitive flexibility. Many activities emphasize both executive functioning and academic skills.[170]

Classifying the comparison condition. The studies cited as evidence for this guide compared the math achievement of children who were exposed to a particular intervention (the intervention group) to the math achievement of children who were not exposed to the intervention of interest (comparison

group). The effectiveness of an intervention must be assessed in the context of a specific comparison. For example, a finding based on an intervention group that received math instruction and a comparison group that received reading instruction concerns the effect of both the math content provided and how it was taught. A finding based on a comparison between intervention children taught math using manipulatives and comparison children taught math without manipulatives concerns the effect of manipulatives.

The panel prioritized the comparison that was most relevant to each recommendation. Thus, studies may have one comparison that appears in one recommendation and a different comparison that appears in a different recommendation.[171] The panel refers to the comparison condition as "regular classroom instruction" when the intervention was offered either as a supplement to standard curriculum or as a replacement for the standard curriculum. In these cases, the comparison group received what the intervention group would have received as part of the regular classroom instruction. In other cases, children exposed to the intervention were compared to children receiving a different, well-defined intervention, which the panel refers to as a "treated comparison." The panel provides the information that was available regarding any curricula used in the comparison condition.

In addition to prioritizing comparisons, the panel assessed the strength of the contrast: the degree to which the instruction the intervention group received differed on key components of the recommendation from the instruction the comparison group received. The panel classified contrasts into three types. In the first type, the intervention group received the key components of the recommendation, and the comparison group did not. In the second type, the panel was able to determine that both the intervention and comparison groups received the key components of the recommendation. An example is a comparison between two curricula that both taught number and operations using a developmental progression in which the only difference between the two groups was the specific curriculum used. The third type included comparisons for which there was incomplete information on the comparison condition, in which case the comparison group may or may not have received the key components of the recommendation. This was the case for studies indicating that the comparison group received "regular classroom instruction" without naming the regular curriculum or providing sufficient detail to support a determination of what instruction the comparison group received. The panel encourages readers to use both the summary of effects and the strength of the contrast to determine the strength of the evidence for a particular study.

Recommendation 1: Teach number and operations using a developmental progression.

Level of evidence: Moderate Evidence

The panel assigned a rating of *moderate evidence* to this recommendation based on their expertise and 21 randomized controlled trials[172] and 2 quasi-experimental studies[173] that met WWC standards and examined interventions including one or more components of Recommendation 1 (see Tables D.2–D.4). The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms. All but two of the studies were conducted in the United States.[174] Positive effects were found in all six outcome domains;[175] however, there were two studies with negative effects,[176] and nine studies reported no discernible effects at least once.[177]

The panel believes that the most effective implementation of Recommendation 1 involves targeted instruction in number and operations according to a developmental progression. Each of the 23 studies included targeted instruction in number and operations for the intervention group, and in 12 cases, the panel

confirmed that the instruction was guided by a developmental progression.[178] The panel did not identify any evidence for the effects of teaching number and operations based on any particular developmental progression. Such a study would have taught the same content to the intervention and comparison groups. The difference would have been the order in which the content was taught, with the intervention group receiving instruction based on a specific developmental progression and the comparison group receiving instruction in the same content in a different order. Based on their expertise and the positive effects found for interventions based on a developmental progression when compared to instruction that does not appear to be based on a developmental progression, the panel recommends the use of a developmental progression to guide instruction. Additional research is needed to identify the developmental progression that reflects how most children learn math.

The panel focused on two characteristics of the research that could limit how well the evidence supported Recommendation 1: (1) whether targeted instruction in number and operations was provided in conjunction with practices addressed in other recommendations (i.e., a multi-component intervention), and (2) the distinction between the number and operations instruction the intervention and comparison groups received. These concerns each made it difficult for the panel to determine the extent to which teaching number and operations using a developmental progression was responsible for the effects seen in math achievement.

The effects of the interventions examined in these 23 studies could not be solely attributed to Recommendation 1, as the interventions were multi-component interventions and included elements of other recommendations.[179] For example, the majority of the studies examined interventions that dealt with early math broadly and included instruction in early math content areas beyond number and operations (i.e., those listed in Recommendation 2). Additionally, studies examining whole curricula such as *Building Blocks*[180] also included core components of Recommendations 3, 4, and 5. As a result, the panel was unable to isolate the effects of instruction in number and operations. Without studies providing an isolated (or direct) test of this recommendation, it is impossible to say conclusively that the causes of the effects seen are the result of practices aligned with the panel's suggestions of how to implement this recommendation. However, in the panel's estimation, teaching number and operations was a primary component of many of the interventions that showed positive effects.

Likewise, although many of the interventions that comprised the evidence for Recommendation 1 were informed by a developmental progression, no study specifically examined how a teacher's use of a developmental progression affected children's performance on math assessments compared with children who might be taught similar content by a teacher not following a developmental progression. Thus, despite the relatively large body of evidence that supports this recommendation, the lack of a direct test of the developmental progression prevented the panel from assigning a strong rating to this recommendation.

The panel also considered the differences between the intervention and comparison groups (or the strength of the contrast) in assigning the level of evidence rating. Although the intervention group in all 23 studies in this body of evidence incorporated targeted instruction in number and operations, in 9 cases the panel determined that the comparison group received similar instruction.[181] The panel determined the intervention in 8 of the 23 studies included instruction in number and operations that was supplemental in nature—that is, offered in addition to regular classroom instruction in math.[182] Findings for these studies included positive effects,[183] no discernible effects,[184] and negative effects.[185]

Instruction for the comparison group was clearly identified for all comparison children in 5 of the 23 studies.[186] The panel determined that, in these five studies, the comparison

children also received targeted instruction in number and operations; however, these children may not have received the same amount of targeted instruction in number and operations or may not have received instruction in which a developmental progression shaped the sequence in which number and operations topics were introduced. Findings in these five studies were positive in the domains of basic number concepts, geometry, and general numeracy; mixed findings were reported in the domain of operations.[187]

In 10 of the 23 studies, the panel believes the comparison group, or some portion of the comparison group, may have received targeted instruction in number and operations and/or received instruction based on a developmental progression.[188] This conclusion is based on the inability to definitively determine the presence, and nature, of number and operations instruction for all members of the comparison group based on the information provided. Findings in these 10 studies were positive[189] or no discernible[190] effects.

The panel concluded that there is a strong pattern of positive effects on children's early math achievement across a range of curricula with a focus on number and operations, even in studies in which the comparison group also received instruction in number and operations.[191] Thus, although there are few studies that directly test the effect of instruction in number and operations using a developmental progression to guide instruction,[192] there is sufficient evidence to warrant a level of evidence rating of moderate.

The 23 studies that contribute to the level of evidence rating for this recommendation include targeted instruction in number and operations, sometimes guided by a developmental progression. However, the intent of the curriculum differs. The panel used their expertise to classify the curricula based on their intent to facilitate a more detailed discussion of the body of evidence. The panel identified three types of curricula represented in the studies: (1) early math curricula, which focus on math including number and operations as well as other early math content areas such as geometry, patterns, measurement, and data analysis; (2) comprehensive early childhood curricula with an explicit math component, which include math instruction as well as instruction in other areas such as literacy; and (3) targeted math interventions, which focus on a particular early math skill. Each type of curricula is discussed in greater detail below.

Early math curricula. The panel reviewed seven studies describing four different curricula with a focus on multiple early math content areas (see Table D.2).[193] Three of the four curricula taught number and operations using a developmental progression to guide instruction.[194] The panel determined that in three of the seven studies, the comparison group received instruction in number and operations.[195] In the remaining studies,[196] the panel concluded that the comparison group may have received instruction in number and operations, as that is a frequently taught early math content area.[197] Among the studies reviewed, consistent positive effects were found for these curricula, particularly in the domains of basic number concepts and general numeracy.[198]

Building Blocks was examined as a stand-alone curriculum in three studies[199] and in combination with the *Pre-K Mathematics* curriculum in one study.[200] The number and operations component of *Building Blocks* includes counting; comparing numbers; number recognition and subitizing; composing and decomposing numbers; and addition, subtraction, multiplication, and division. Three studies that examined the *Building Blocks* curriculum with primarily low-income urban children found a positive effect for, or improvement in, the domains of general numeracy and basic number concepts, when compared with regular classroom instruction, including classrooms in which number and operations was a part of the curriculum being used.[201] In a fourth study that combined the computer-based activities of *Building Blocks* with teacher-directed activities from the *Pre-K Mathematics* curriculum,

researchers found a positive effect on children's performance in general numeracy when compared to children receiving regular classroom instruction.[202]

One study examined the classroom and home components of the *Pre-K Mathematics* curriculum combined with the software program from *DLM Early Childhood Express*.[203] Classrooms were randomly assigned to receive the intervention (*Pre-K Mathematics* and *DLM Early Childhood Express*) or regular classroom instruction, which included *Creative Curriculum*, a comprehensive early childhood education curriculum used to teach number and operations that uses a developmental progression to inform instruction. Children in classrooms using the *Pre-K Mathematics* curriculum and *DLM Early Childhood Express* scored higher in the general numeracy domain, on average, than children whose teachers taught them math using regular classroom instruction.

Two curricula that focus narrowly on number and operations but are comprehensive in the aspects of number and operations addressed are *Rightstart* and *Math Is Everywhere*. *Rightstart* is grounded in a theoretical model that children must be taught central numerical concepts before learning more formal math skills. In a study of *Rightstart*, children who received instruction using the *Rightstart* curriculum improved their performance in the domain of basic number concepts, when compared to children receiving regular classroom instruction.[204] *Math Is Everywhere* is a curriculum designed to fully integrate math into regular classroom practice. Although there is no clear developmental progression for *Math Is Everywhere*, it is comprehensive in the number and operations concepts addressed—including counting, number recognition, one-to-one correspondence, comparison, operations, and quantity understanding. In a study of *Math Is Everywhere*, children who received the *Math Is Everywhere* curriculum performed better on average on a test of general numeracy skills than children in a comparison condition who received regular classroom instruction.[205]

Table D.2. Studies of early math curricula that taught number and operations and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Arnold et al. (2002)[5] RCT Meets evidence standards without reservations	Pairs of half-day or full-day Head Start classes Children: 103 total (49 intervention; 54 comparison) Age range: 3.1 to 5.3 years Average age: 4.4 years (SD 7.32 months)	*Math Is Everywhere* vs. regular classroom instruction	General numeracy: TEMA-2 Positive (0.40, ns)	?	
Clements and Sarama (2007b)[5,6] RCT Meets evidence standards with reservations	Preschool classrooms in state-funded or Head Start programs Children: 68 total (30 intervention; 38 comparison) Age range: 2.9 to 4.8 years Mean age: 4.2 years (SD 6.2 months)	*Building Blocks* vs. regular classroom instruction (*Creative Curriculum*, locally developed)	Basic number concepts: BB Assessment–Number Scale Positive (0.75*)	X[7]	X[7]
			Geometry: BB Assessment–Geometry Scale Positive (1.40*)	X[7]	X[7]

(continued)

Table D.2. Studies of early math curricula that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Clements and Sarama (2008)[5,8] RCT Meets evidence standards without reservations	24 teachers in Head Start or state-funded preschool programs were randomly assigned to one of three conditions. 20 teachers in programs serving low- and middle- income students were randomly assigned to one of two conditions. Children: 201 total (101 intervention; 100 comparison) Age range: Children had to be within kindergarten entry range for the following year.	*Building Blocks* vs. regular classroom instruction (locally developed)	General numeracy: REMA Positive (1.07*)	?	?
Clements et al. (2011)[5,9,10] RCT Meets evidence standards without reservations	Prekindergarten classrooms in two urban public school districts Children: 1,305 total (927 intervention; 378 comparison)	*Building Blocks* vs. regular classroom instruction (*Where Bright Futures Begin*; *Opening the World of Learning*; *Investigations in Number, Data, and Space*; *DLM Early Childhood Express*)	General numeracy: REMA–Total Score Positive (0.48*)	X[11]	X[11]
			Basic number concepts: REMA–Numbers Total Score Positive (0.39*)	X[11]	X[11]
			Geometry: REMA–Geometry Total Score Positive (0.64*)	X[11]	X[11]
Griffin, Case, and Capodilupo (1995) and related publication Griffin, Case, and Siegler (1994)[12] QED Meets evidence standards with reservations	Kindergarten students in public schools in inner-city areas in Massachusetts Children: 47 total (23 intervention; 24 comparison)	*Rightstart* vs. regular classroom instruction	Basic number concepts: NKT Positive (1.79*)	?	?
Klein et al. (2008)[5] RCT Meets evidence standards without reservations	40 prekindergarten classrooms in Head Start or state-funded programs in New York and California Children: 278 total (138 intervention; 140 comparison) Age range: 3.8 to 4.9 years Mean age: 4.4 years	*Pre-K Mathematics* combined with *DLM Early Childhood Express* vs. regular classroom instruction (*Creative Curriculum*, *High Scope*, Montessori, locally developed)	General numeracy: CMA Positive (0.57*)	X[13]	X[13]

(continued)

Table D.2. Studies of early math curricula that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Sarama et al. (2008)[14] RCT Meets evidence standards without reservations	Head Start or state-funded prekindergarten classrooms in New York and California Children: 200 total (104 intervention; 96 comparison) Average age: 4.3 years	*Building Blocks* combined with *Pre-K Mathematics* vs. regular classroom instruction	General numeracy: REMA Positive (0.62*)	?	?

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that taught number and operations and that may have used a developmental progression to guide that instruction.

X The intervention included this component.

BB Assessment = Building Blocks Assessment of Early Mathematics[206]
REMA = Research-Based Early Math Assessment[207]
CMA = Child Math Assessment[208]
TEMA-2 = Test of Early Mathematics Ability, second edition[209]
NKT = Number Knowledge Test[210]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum or curricula the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" refers to findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] **Clements and Sarama (2007b)** also reported scores for the subscales of the Numbers and Geometry scales; positive effects were seen for each subscale. Findings from **Clements and Sarama (2007b)** were previously reported in the WWC intervention report on *SRA Real Math Building Blocks PreK*. The panel reports the same findings as presented in the intervention report.

[7] In **Clements and Sarama (2007b)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *Building Blocks* and the curricula used in the comparison classrooms, including the branded comprehensive early childhood curriculum *Creative Curriculum*. The intervention group participated in *Building Blocks*, a math curriculum that included instruction in number and operations guided by a developmental progression. The comparison group participated in a variety of curricula, including *Creative Curriculum*, which also included instruction in number and operations guided by a developmental progression.

[8] For **Clements and Sarama (2008)**, the WWC is reporting author-reported effect sizes consistent with prior reporting of findings from this study in the WWC intervention report on *SRA Real Math Building Blocks PreK*.

[9] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[10] **Clements et al. (2011)** also reported the subscale scores from the REMA. Findings for the subscale scores were consistent with the total score findings and generally positive (9 of 13 scores). No discernible effects were seen for 4 of the 13 subscale scores (two in the geometry domain: transformations/turns and comparing shapes; one in the operations domain: arithmetic; and one in the basic number concepts domain: composition of number).

[11] **In Clements et al. (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *Building Blocks* and the various branded curricula used in the comparison classrooms, including *DLM Early Childhood Express*, a comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that included instruction in number and operations guided by a developmental progression. The comparison group participated in a

number of branded curricula, including *DLM Early Childhood Express*, an early childhood curriculum that included instruction in number and operations but was not guided by a developmental progression in the same manner as *Building Blocks* instruction.

[12] **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)** reported other outcomes for which no pretest data were provided. The WWC was unable to conduct a review that included these outcomes, as baseline equivalence could not be established.

[13] In Klein et al. (2008), the difference between the intervention and comparison groups included any aspect of instruction that differed between the combined *Pre-K Mathematics* curriculum and *DLM Early Childhood Express* intervention and the curricula used in the comparison classrooms, including the branded comprehensive early childhood curriculum *Creative Curriculum*. The intervention group, which participated in a combination of *Pre-K Mathematics* curriculum and *DLM Early Childhood Express*, included instruction in number and operations using a developmental progression. The comparison group participated in a number of branded curricula, including *Creative Curriculum*, a comprehensive early childhood curriculum that included instruction in number and operations guided by a developmental progression.

[14] **Sarama et al. (2008)** reported subscale scores as well; however, only the means were provided, so the WWC was unable to calculate effect sizes for the subscales.

Comprehensive curricula with an explicit math component. Classrooms may also use curricula that include more than just math—for example, a curriculum that includes math, reading, and science in a single package. The panel reviewed five studies of comprehensive curricula that included an explicit math component (see Table D.3).[211] Each study compared a specific comprehensive curriculum to regular classroom instruction, which may have included instruction in number and operations. The studies demonstrated mixed findings, with three studies of broader curricula showing no discernible effects on children's math achievement,[212] one study demonstrating both positive and no discernible effects in basic number concepts and geometry for a curriculum aimed at developing children's problem-solving skills,[213] and a final study finding positive effects on general numeracy.[214]

The three curricula that demonstrated no discernible effects in math outcomes focused on either literacy or self-regulation skills but included a math component. The *Bright Beginnings* and *Creative Curriculum* programs are comprehensive curricula, with an emphasis on literacy, that also include math units. It was not clear from the studies reviewed how much time teachers were encouraged to devote to math; however, there was a clear intent for the curricula to support math instruction. Evaluations of the curricula found no discernible effects on children's math outcomes.[215] *Tools of the Mind* is a comprehensive curriculum that focuses on improving children's self-regulation skills. The goal is to promote children's abilities to regulate their own behavior to increase social and academic skills. A portion of *Tools of the Mind* focuses on math instruction and includes counting, one-to-one correspondence, patterns, and number recognition. A study of *Tools of the Mind* found no discernible effects in math compared with a literacy-focused comparison condition, but comparison children may have also received some math instruction.[216]

In another study, two more narrowly focused curricula were combined, resulting in a curriculum with a focus on metacognitive skills to promote problem solving.[217] An evaluation of the combined *Let's Think/Maths!* curricula demonstrated positive effects for children's basic number concepts at immediate posttest and maintenance (six months). Positive results were found in the geometry outcome domain at posttest, but no discernible effects were seen at maintenance (six months).

The *Evidence-based Program for Integrated Curricula (EPIC)* incorporates math, language, literacy, and learning behaviors in a developmentally grounded approach to preschool instruction. The math component of the curriculum follows a developmental scope and sequence and covers number knowledge, sorting, comparison, shapes, measurement, and addition and subtraction. A study of the effectiveness of *EPIC* reported positive effects on children's general numeracy knowledge, compared to children who participated in regular classroom instruction using *DLM Early Childhood Express*, which is known to include instruction on number and operations but does not use developmental progressions to guide instruction in the same manner.[218]

Table D.3. Studies of comprehensive curricula with an explicit math component that taught number and operations and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Aunio, Hautamaki, and Van Luit (2005)[5,6] RCT Meets evidence standards without reservations	Pairs of matched students attending two large preschool in Helsinki, Finland were randomly assigned. Four smaller preschools in Helsinki, Finland were randomly assigned. Children: 45 total (22 intervention; 23 comparison) Age range: 4.7 to 6.6 years Mean age: 5.5 years (SD 6.4 months)	*Let's Think!* combined with *Maths!* vs. regular classroom instruction	Basic number concepts: ENT–Relational Scale, Posttest Positive (0.77, ns)	?	
			Basic number concepts: ENT–Counting Scale, Posttest Positive (0.87, ns)	?	
			Geometry: Geometrical Analogies, Posttest Positive (0.25, ns)	?	
			Geometry: SRT–Posttest No discernible (0.20, ns)	?	
			Basic number concepts: ENT–Relational Scale, Maintenance (6 months) Positive (0.48, ns)	?	
			Basic number concepts: ENT–Counting Scale, Maintenance (6 months) Positive (0.36, ns)	?	
			Geometry: Geometrical Analogies, Maintenance (6 months) No discernible (0.24, ns)	?	
			Geometry: SRT, Maintenance (6 months) Positive (0.36, ns)	?	
Barnett et al. (2008) RCT Meets evidence standards with reservations	Children attending a full-day preschool program Children: 202 total (85 intervention; 117 comparison) Age range: 3 to 4 years; slightly more 4-year-olds (54%)	*Tools of the Mind* vs. regular classroom instruction (district-created, balanced literacy)	Operations: WJ-Revised–Applied Math Problems Subtest No discernible (0.17, ns)	X[7]	X[7]
Fantuzzo, Gadsden, and McDermott (2011)[8] RCT Meets evidence standards without reservations	80 Head Start classrooms in Philadelphia, Pennsylvania Children: 778 total (397 intervention; 381 comparison) Age range: 2.9 to 5.8 years. Mean age: 4.2 years (SD 6.8 months)	*Evidence-based Program for Integrated Curricula (EPIC)* vs. regular classroom instruction (*DLM Early Childhood Express*)	General numeracy: LE–Mathematics, Wave 4 Positive (0.18*)	X[9]	X[9]

(continued)

Table D.3. Studies of comprehensive curricula with an explicit math component that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
PCER Consortium (2008, Chapter 2)[5,10] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs were randomly assigned the year before the study began. Children: 193 total (93 intervention; 100 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.17, ns)	?	?
			General numeracy: CMA-A, Posttest No discernible (0.10, ns)	?	?
			Geometry: Shape Composition, Posttest No discernible (–0.12, ns)	?	?
PCER Consortium (2008, Chapter 2)[5,10] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs were randomly assigned the year before the study began. Children: 198 total (98 intervention; 100 comparison) Mean age: 4.5 years	*Bright Beginnings* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.16, ns)	?	
			General numeracy: CMA-A, Posttest No discernible (0.14, ns)	?	
			Geometry: Shape Composition, Posttest No discernible (–0.03, ns)	?	
PCER Consortium (2008, Chapter 3)[11] RCT Meets evidence standards with reservations	Preschoolers attending Head Start centers Children: 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.20, ns)	?	?
			General numeracy: CMA-A–Mathematics Composite, Posttest No discernible (–0.10, ns)	?	?
			Geometry: Shape Composition, Posttest No discernible (0.19, ns)	?	?
			Operations: WJ-III–Applied Problems, Maintenance No discernible (0.09, ns)	?	?
			General numeracy: CMA-A–Mathematics Composite, Maintenance No discernible (0.14, ns)	?	?
			Geometry: Shape Composition, Maintenance No discernible (–0.01, ns)	?	?

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that taught number and operations and that may have used a developmental progression to guide that instruction.

X The intervention included this component.

ENT = Early Numeracy Test[219]
SRT = Spatial Relationships Test[220]

CMA-A = Child Math Assessment–Abbreviated[221]
WJ-III = Woodcock-Johnson, third edition[222]
LE = Learning Express[223]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum or curricula the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" refers to findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[7] In **Barnett et al. (2008)**, the difference between the intervention and comparison groups with respect to math instruction is not known. The intervention group participated in *Tools of the Mind*, a comprehensive early childhood curriculum with a math component that supports incorporating math into other parts of the school day. The comparison group participated in a district-created balanced literacy curriculum. From the information provided, it is not clear how the intervention and comparison groups differed with respect to instruction in number and operations or the use of a developmental progression to guide instruction in number and operations.

[8] **Fantuzzo, Gadsden, and McDermott (2011)** reported on four waves of data collection. The panel decided to use Wave 1 as pretest data, because it was collected prior to the delivery of math content. Wave 4 was used as the posttest, as it was collected at the end of the school year and delivery of the intervention. Waves 2 and 3 could be viewed as intermediary outcomes, but the panel chose to focus on posttests when determining levels of evidence.

[9] In **Fantuzzo, Gadsden, and McDermott (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *EPIC* and *DLM Early Childhood Express*, a branded comprehensive early childhood curriculum. The intervention group participated in *EPIC*, a comprehensive early childhood curriculum that included instruction in number and operations guided by a developmental progression. The comparison group participated in another branded comprehensive early childhood curriculum, *DLM Early Childhood Express*, which included number and operations content but was not guided by a developmental progression in the same manner as instruction using *EPIC*.

[10] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel rated the study differently but reports the same findings as presented in the intervention report. The difference in study rating is due to the use of WWC Version 2.1 standards as opposed to WWC Version 1.0 standards. Findings from this study of *Bright Beginnings* were previously reported in the WWC intervention report on *Bright Beginnings*. The panel reports the same findings as reported in the intervention report. For both *Creative Curriculum* and *Bright Beginnings*, the authors report on additional outcomes that were assessed in the spring of kindergarten.

[11] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel reports the same findings as presented in the intervention report.

Targeted interventions. In addition to studies examining commercially available curricula, the panel reviewed 11 studies examining the effectiveness of researcher-developed curricula or interventions focusing on specific math skills, in contrast to the more general math focus in the comprehensive curricula of studies discussed previously. The studies reviewed by the panel described interventions for increasing children's skills in the domains of basic number concepts and operations through specific one-on-one or small-group activities or board games (see Table D.4).[224] Findings were mixed, with positive, negative, and no discernible effects in various areas of children's math achievement.

Number sense skills and knowledge were the focus of six studies.[225] Two studies examined the same curriculum, which was designed for children at risk for difficulties in math.[226] The curriculum included practicing skills

such as number recognition, oral counting, number sequencing, verbal subitizing, and counting on fingers. Results indicated that the supplemental curriculum produced a positive effect on children's general numeracy and operations skills, whether the comparison was with regular classroom instruction or with a treated comparison group that participated in a supplemental language intervention. Some of these effects were maintained at a follow-up.[227]

A second number sense curriculum focused particularly on spatial relations, the understanding of more and less, teaching children to use the numbers five and ten as benchmarks when making comparisons of quantities, and understanding that numbers are made up of multiple other numbers (i.e., part-whole relations).[228] The curriculum replaced 20 minutes per day of district-mandated curriculum. In the study reviewed, compared with children receiving regular classroom instruction, children who participated in the number sense curriculum had higher scores in basic number concepts and operations at immediate posttest. Three weeks after the intervention, effects were maintained in a majority of the outcomes measured.[229]

One study focused on alternative ways to deliver number sense content to low-income children.[230] Three intervention groups received the same instruction in number lines, cardinality, counting, comparison, and addition and subtraction. One group, the math-only group, received this instruction through traditional small-group instruction. The math-with-story group received the same content but read stories together as the key instructional method. A third group, math with movement, received the same content, but their instruction included movements such as clapping and jumping. The no-math comparison group spent a similar amount of time with the teacher-researcher, reading books but not receiving the number sense instruction. The study found mixed effects when each intervention group was compared to the no-math comparison group. For the math-only and math-with-story groups, there was no difference in the general numeracy skills of the intervention children compared with the no-math comparison group. However, children who participated in the math-with-movement group scored higher on a general numeracy outcome than children in the no-math comparison group.

Another researcher-developed curriculum focused on units of quantification and the application of these units to counting and reasoning about numerical increases and decreases, measurement, and relations among geometric shapes.[231] The number-focused activities focused on skills including the counting sequence, subtraction by backward counting, increasing and decreasing numerical quantity, inverse relations between unit size and numerical measure, inverse relations in addition and subtraction, and one-to-one correspondence. In the study reviewed, children who participated in the curriculum scored, on average, higher on general numeracy outcomes than children who participated in the comparison group (literacy instruction).

A researcher-developed curriculum focused on instruction in numbers that was playful and self-directed.[232] Children in the numeracy instruction group used toy ponies and foam numbers to learn numbers from 1 to 10, and then to count from 1 to 10 using blocks. Once children were proficient, they learned additional numbers from 10 to 30 using the same games. On occasion, children also played bingo using the numbers they knew. In the study reviewed, children who participated in the numeracy instruction group were compared to children who participated in additional art instruction or children who participated in a cognitive instruction condition that included additional instruction in the oddity principle, inserting objects into series, and conservation. Children in the numeracy instruction group scored, on average, higher on operations outcomes than children who participated in the art comparison group; however, there were no differences in their performance on outcomes in either the basic number concepts or patterns

and classification domains. Children in the numeracy instruction group scored, on average, lower on outcomes in the basic number concepts, operations, and patterns and classification domains than their classmates who participated in the cognitive instruction.

One study assessed the effectiveness of adults providing counting support to children working on balance-beam tasks related to differences in weight or distance.[233] The first experiment in the study examined four outcomes in the basic number concepts domain and found positive, negative,[234] and no discernible effects. A second experiment in the study examined the effectiveness of providing similar adult counting assistance using four other tasks in basic number concepts, including the balance-beam task. In the second experiment, children who received adult assistance performed better, on average, than children who did not receive such support on two tasks in the basic number concepts domain. There was no difference in performance on the other two tasks, which were also in the basic number concepts domain.

Two studies examined targeted instruction on specific components in the domains of basic number concepts and operations. The first of these studies contrasted three methods of number sense instruction (structured discovery learning, unstructured discovery learning, and structured discovery plus explicit instruction) with a treated comparison of "haphazard practice" of number-after examples.[235] Children who received semi-structured discovery learning or explicit instruction performed better in assessments of some operations outcomes compared with children who participated in haphazard practice. The second study focused on the inversion principle (adding a number can be undone by subtracting the same number).[236] Children who received training on this concept performed an inversion task (an operations outcome) better than children who did not receive the training.

Two studies examined the effectiveness of playing number-based board games compared with playing color-based board games.[237] In the first study, children in both groups played board games one-on-one with an adult. Children in the intervention group who played either linear or circular number-based board games scored higher on measures of basic number concepts and number recognition at posttest and maintenance than children who played a color-based board game.[238] The second study examined the achievement of children playing linear number-based board games with children playing color-based board games. Positive effects were found in basic number concepts, with children playing the linear number-based board games scoring higher than the children in the treated comparison group who played a color-based board game.[239]

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Baroody, Eiland, and Thompson (2009)[6] RCT Meets evidence standards without reservations	Children attending public preschools for at-risk children Children: 40 total (20 intervention; 20 comparison) Age range: 4 to 5.25 years	Semi-structured discovery learning vs. treated comparison (haphazard practice)	Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+0/0+*n* items No discernible (0.06, ns)	X[7]	
			Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+1/1+*n* items Positive (0.55, ns)	X[7]	
Baroody, Eiland, and Thompson (2009)[6] RCT Meets evidence standards without reservations	Children attending public preschools for at-risk children Children: 40 total (20 intervention; 20 comparison) Age range: 4 to 5.25 years	Structured discovery learning vs. treated comparison (haphazard practice)	Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+0/0+*n* items No discernible (0.00, ns)	X[8]	
			Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+1/1+*n* items No discernible (–0.17, ns)	X[8]	
Baroody, Eiland, and Thompson (2009)[6] RCT Meets evidence standards without reservations	Children attending public preschools for at-risk children Children: 40 total (20 intervention; 20 comparison) Age range: 4 to 5.25 years	Structured discovery learning with explicit instruction on patterns/relations vs. treated comparison (haphazard practice)	Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+0/0+*n* items No discernible (0.00, ns)	X[9]	
			Operations: Percentage of children scoring at least 85% accurate (E-3 Scale) for *n*+1/1+*n* items Positive (1.20*)	X[9]	

(continued)

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Curtis, Okamoto, and Weckbacher (2009, Experiment 1)[6,10,11] RCT Meets evidence standards without reservations	Preschoolers attending a university-based program in California Children: 25 total (14 intervention; 21 comparison) Age range: 3 years, 2 months to 5 years Mean age: 4 years	Adult support (adults counted weights or pegs and repeated final number to indicate cardinal value of set) vs. no adult support	Basic number concepts: Balance Beam Scores –Large Difference in Weights Negative (–0.41, ns)	X[12]	
			Basic number concepts: Balance Beam Scores –Small Difference in Weights Positive (0.66, ns)	X[12]	
			Basic number concepts: Balance Beam Scores –Large Difference in Distance No discernible (–0.07, ns)	X[12]	
			Basic number concepts: Balance Beam Scores –Small Difference in Distance No discernible (0.16, ns)	X[12]	
Curtis, Okamoto, and Weckbacher (2009, Experiment 2)[10,13] RCT Meets evidence standards without reservations	Preschoolers attending one of two California preschool programs (including a university-based program) or a preschool in West Virginia Children: 54 total (27 intervention; 27 comparison) Age range: 3 years, 5 months to 4 years, 11 months Mean age: 4 years, 4 months; 4 years, 4 months for the intervention group (SD 5 months); 4 years, 3 months for the comparison group (SD 4 months)	Adult support (adults counted weights or pegs and repeated final number to indicate cardinal value of set) vs. no adult support	Basic number concepts: Stickers on Cards Positive (0.61, ns)	X[12]	
			Basic number concepts: Stacks of Counting Chips No discernible (0.23, ns)	X[12]	
			Basic number concepts: Weights on Balance Scale Positive (0.35, ns)	X[12]	
			Basic number concepts: Distance on Balance Scale No discernible (0.23, ns)	X[12]	

(continued)

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Dyson, Jordan, and Glutting (2013)[10,14] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 121 total (56 intervention; 65 comparison) Mean age: 5.5 years (SD 4.0 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers*)	General numeracy: NSB–Total Score, Posttest Positive (0.64*)	X[15]	X[15]
			Operations: WJ-III–Total Score, Posttest Positive (0.29, ns)	X[15]	X[15]
			General numeracy: NSB–Total Score, Maintenance (6 weeks) Positive (0.65*)	X[15]	X[15]
			Operations: WJ-III–Total Score, Maintenance (6 weeks) No discernible (0.18, ns)	X[15]	X[15]
Jordan et al. (2012)[10,16] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 86 total (42 intervention; 44 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers* or *Math Connects*)	General numeracy: NSB–Total, Posttest Positive (1.10*)	X[17]	X[17]
			Operations: WJ-III–Total, Posttest Positive (0.91*)	X[17]	X[17]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.77*)	X[17]	X[17]
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.56*)	X[17]	X[17]
Jordan et al. (2012)[10,16] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 84 total (42 intervention; 42 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. treated comparison (supplemental language intervention with *Math Trailblazers* or *Math Connects*)	General numeracy: NSB–Total, Posttest Positive (0.91*)	X[18]	X[18]
			Operations: WJ-III–Total, Posttest Positive (0.84*)	X[18]	X[18]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.62*)	X[18]	X[18]
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.75*)	X[18]	X[18]

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Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Kidd et al. (2008)[10] RCT Meets evidence standards without reservations	Children attending kindergarten in a metropolitan school district Children: 52 total (26 intervention; 26 comparison) Age: All were 5 years old by the end of September the year the intervention was implemented.	Numeracy vs. treated comparison (art)	Basic number concepts: Conservation Test No discernible (–0.11, ns)	X[19]	
			Operations: WJ-III Applied Problems Positive (0.31, ns)	X[19]	
			Patterns and classification: Oddity Test No discernible (0.04, ns)	X[19]	
			Patterns and classification: OLSAT Classification Scale No discernible (–0.03, ns)	X[19]	
			Patterns and classification: Seriation Test No discernible (0.10, ns)	X[19]	
Kidd et al. (2008)[10] RCT Meets evidence standards without reservations	Children attending kindergarten in a metropolitan school district Children: 52 total (26 intervention; 26 comparison) Age: All were 5 years old by the end of September the year the intervention was implemented.	Numeracy vs. treated comparison (cognitive instruction in oddity principle, inserting objects into series, and conservation)	Basic number concepts: Conservation Test Negative (–0.68*)	X[20]	
			Operations: WJ-III Applied Problems Negative (–0.50, ns)	X[20]	
			Patterns and classification: Oddity Test Negative (–0.68*)	X[20]	
			Patterns and classification: OLSAT Classification Scale Negative (–0.46, ns)	X[20]	
			Patterns and classification: Seriation Test Negative (–0.64, ns)	X[20]	
Lai, Baroody, and Johnson (2008) RCT Meets evidence standards without reservations	Children attending middle-SES public school in Taoyuan and lower-SES public school in Dayuan Children: 30 total (15 intervention; 15 comparison) Age range: 4 to 5 years	Inverse training vs. treated comparison (decomposition/composition)	Operations: Gain in Performance on Inversion Trials Positive (0.54*)	X[21]	

(continued)

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Monahan (2007)[8,22] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 80 total (41 intervention; 39 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math vs. treated comparison (reading books)	General numeracy: ENCO Assessment No discernible (0.02, ns)	X[23]	
Monahan (2007)[8,22] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 81 total (42 intervention; 39 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math with story vs. treated comparison (reading books)	General numeracy: ENCO Assessment No discernible (0.00, ns)	X[24]	
Monahan (2007)[8,22] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 78 total (39 intervention; 39 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math with movement vs. treated comparison (reading books)	General numeracy: ENCO Assessment Positive (0.32, ns)	X[25]	
Ramani and Siegler (2008)[10,26] RCT Meets evidence standards without reservations	Preschoolers attending Head Start programs Children: 124 total (68 intervention; 56 comparison) Age range: 4 years, 1 month to 5 years, 5 months Mean age: 4 years, 9 months (SD 0.44)	Number-based board games vs. treated comparison (color-based board games)	Basic number concepts: Counting, Posttest Positive (0.74*)	X[27]	
			Basic number concepts: Numerical Magnitude Comparison, Posttest Positive (0.99*)	X[27]	
			Number recognition: Number Identification, Posttest Positive (0.69*)	X[27]	
			Basic number concepts: Counting, Maintenance (9 weeks) Positive (0.66*)	X[27]	
			Basic number concepts: Numerical Magnitude Comparison, Maintenance (9 weeks) Positive (0.77*)	X[27]	
			Number recognition: Number Identification, Maintenance (9 weeks) Positive (0.80*)	X[27]	

(continued)

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Siegler and Ramani (2008, Experiment 2)[10,26] RCT Meets evidence standards without reservations	Preschool-aged children attending Head Start or one of three childcare centers Children: 36 total (18 intervention; 18 comparison) Age range: 4 to 5.1 years Mean age: 4.6 years (SD 0.30) for the linear number-based board games group; 4.7 years (SD 0.42) for the color-based board games group	Linear, number-based board games vs. treated comparison (color-based board games)	Basic number concepts: Number Line Estimation–Percent Absolute Error Positive (0.86*)[25]	X[28]	
			Basic number concepts: Percent of Correctly Ordered Numbers Positive (1.17*)	X[28]	
Sood (2009)[5,10] RCT Meets evidence standards with reservations	Kindergarten classrooms in an urban elementary school in Pennsylvania Children: 101 total (61 intervention; 40 comparison) Mean age: 5.4 years (SD 4.32 months) for the intervention group; 5.6 years (SD 3.90 months) for the control group	Researcher-developed number sense curriculum vs. regular classroom instruction (district-mandated curriculum)	Basic number concepts: Oral Counting Fluency, Posttest No discernible (0.09, ns)	?	
			Basic number concepts: Counting From, Posttest Positive (0.28, ns)	?	
			Number recognition: Number Identification, Posttest Positive (0.33, ns)	?	
			Patterns and classification: Spatial Relationships, Posttest Positive (0.58, ns)	?	
			Basic number concepts: Number Relationships, Posttest Positive (1.23*)	?	
			Basic number concepts: Five and Ten Frame Identification and Representation, Posttest Positive (0.92, ns)	?	
			Operations: Five and Ten Frame Calculations, Posttest Positive (0.60, ns)	?	
			Operations: Nonverbal Calculations, Posttest Positive (0.37, ns)	?	
			Basic number concepts: Oral Counting Fluency, Maintenance (3 weeks) No discernible (0.12, ns)	?	

(continued)

Table D.4. Studies of targeted interventions that taught number and operations and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Number and Operations Skills**	**Use a Developmental Progression**
Sood (2009)[5,10] RCT Meets evidence standards with reservations *(continued)*	Kindergarten classrooms in an urban elementary school in Pennsylvania Children: 101 total (61 intervention; 40 comparison) Mean age: 5.4 years (SD 4.32 months) for the intervention group; 5.6 years (SD 3.90 months) for the comparison group	Researcher-developed number sense curriculum vs. regular classroom instruction (district-mandated curriculum)	Basic number concepts: Counting From, Maintenance (3 weeks) No discernible (0.18, ns)	?	
			Number recognition: Number Identification, Maintenance (3 weeks) No discernible (–0.06, ns)	?	
			Patterns and classification: Spatial Relationships, Maintenance (3 weeks) Positive (1.09*)	?	
			Basic number concepts: Number Relationships, Maintenance (3 weeks) Positive (0.65, ns)	?	
			Basic number concepts: Five and Ten Frame Identification and Representation, Maintenance (3 weeks) Positive (1.19*)	?	
			Operations: Five and Ten Frame Calculations, Maintenance (3 weeks) Positive (0.92, ns)	?	
			Operations: Nonverbal Calculations, Maintenance (3 weeks) No discernible (0.12, ns)	?	
Sophian (2004)[5,10] QED Meets evidence standards with reservations	Head Start sites Children: 94 total (46 intervention; 48 comparison) Age range: 2 years, 6 months to 4 years, 7 months	Researcher-developed measurement-focused curriculum vs. treated comparison (literacy instruction)	General numeracy: DSC–Mathematics Subscale Positive (0.33, ns)	X[29]	

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that taught number and operations and that may have used a developmental progression to guide that instruction.

X The intervention included this component.

ENCO = Emergent Numeracy and Cultural Orientations Assessment[240]

NSB = Number Sense Brief[241]

OLSAT = Otis-Lennon School Ability Test[242]

WJ-III = Woodcock-Johnson, third edition[243]

DSC = Developing Skills Checklist[244]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum or curricula the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" refers to findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] **Baroody, Eiland, and Thompson (2009)** reported six different scoring methods as well as the TEMA. The panel selected the E-3 scale, which excluded answers the child determined by counting (verbal, finger, or object counting) and excluded response biases (i.e., nonselective application of a strategy that was used on more than half the items and that did not make sense for at least one of the items). The TEMA was not reported for a comparison of interest to the panel and thus was not included in the review.

[7] In **Baroody, Eiland, and Thompson (2009)**, there were three intervention groups that could be compared with a single comparison group. In this contrast, the difference between the intervention and comparison groups was in the manner of presentation of the same number and operations material (number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations). Both the intervention and comparison groups participated in a core manipulative–based and game-based curriculum that developed the prerequisites for mental addition. During the second phase, all groups used a computer-supported curriculum to promote mastery of addition and estimation skills, although the nature of the curriculum differed. The intervention group participated in a semi-structured, computer-supported discovery-learning condition. Children practiced number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations in four blocks of five items. The comparison group had haphazard practice of the same four types of items (number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations).

[8] In **Baroody, Eiland, and Thompson (2009)**, there were three intervention groups that could be compared with a single comparison group. In this contrast, the difference between the intervention and comparison groups was in the manner of presentation of the same number and operations material (number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations). Both the intervention and comparison groups participated in a core manipulative–based and game-based curriculum that developed the prerequisites for mental addition. During the second phase, all groups used a computer-supported curriculum to promote mastery of addition and estimation skills, although the nature of the curriculum differed. The intervention group practiced in an implicitly structured discovery-learning manner. Children practiced three items consecutively to highlight relations between number-after, related *n*+1/1+*n* combinations, and related *n*+0/0+*n* facts. The comparison group had haphazard practice of the same four types of items (number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations).

[9] In **Baroody, Eiland, and Thompson (2009)**, there were three intervention groups that could be compared with a single comparison group. In this contrast, the difference between the intervention and comparison groups was in the manner of presentation of the same number and operations material (number-after, *n*+0/0+*n* facts, *n*+1/1+n items, and other combinations). Both the intervention and comparison groups participated in a core manipulative–based and game-based curriculum that developed the prerequisites for mental addition. During the second phase, all groups used a computer-supported curriculum to promote mastery of addition and estimation skills, although the nature of the curriculum differed. The intervention group practiced in an explicitly structured discovery-learning manner. Adults provided explicit instruction (i.e., "When we add one, it's just the number after the other number"), while children practiced three items consecutively to highlight relations between number-after, related *n*+1/1+*n* combinations, and related *n*+0/0+*n* facts. The comparison group had haphazard practice of the same four types of items (number-after, *n*+0/0+*n* facts, *n*+1/1+*n* items, and other combinations).

[10] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[11] For Experiment 1 in **Curtis, Okamoto, and Weckbacher (2009)**, the panel decided to use the counting outcome as the pretest for the post-hoc difference-in-difference adjustments. There was no pretest for the specific outcomes, but the counting measure was deemed an acceptable substitute by the panel.

[12] In both experiments in **Curtis, Okamoto, and Weckbacher (2009)**, the difference between the intervention and comparison groups in teaching number and operations was whether the children received adult support in counting items. Children in the intervention group completed math tasks with an adult pointing to and counting aloud the number of items with repetition of the final number to reinforce the cardinality of the set. The comparison group did not receive assistance from the adult in counting or determining cardinality of the set.

[13] Experiment 2 in **Curtis, Okamoto, and Weckbacher (2009)** did not report pretest data for the outcomes. The panel decided to use the quantity estimation pretest in the post-hoc difference-in-difference adjustments.

[14] **Dyson, Jordan, and Glutting (2013)** reported total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all subscales at posttest and maintenance, except for the WJ-III–Applied Problems subscale, for which no discernible effects were seen at posttest or maintenance.

[15] In **Dyson, Jordan, and Glutting (2013)**, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, generally 3 a week, for a total of 24 sessions (or 12 hours). The sessions included instruction in number and operations that was based on a developmental progression. The comparison group did not receive this additional instruction; rather, they received only regular classroom math instruction.

The regular classroom math instruction, for both the intervention and comparison children, was *Math Trailblazers*, a branded math curriculum used to teach number and operations but not guided by a developmental progression.

[16] **Jordan et al. (2012)** reported posttest and maintenance effects for total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all but seven of the NSB outcomes, which were reported as no discernible effects.

[17] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The sessions included instruction in number and operations that was based on a developmental progression. The comparison group did not receive this additional instruction in math; rather, they received only regular classroom instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*. Both of these are commercially available curricula that teach number and operations; the panel determined that *Math Trailblazers* does not use a developmental progression to guide instruction.

[18] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The sessions included instruction in number and operations that was based on a developmental progression. The comparison group did not receive this additional instruction in math; rather, they only received regular classroom instruction and additional literacy instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*. Both of these are commercially available curricula that teach number and operations; the panel determined that *Math Trailblazers* does not use a developmental progression to guide instruction.

[19] There were two comparisons in **Kidd et al. (2008)**. In this comparison, the difference between the intervention and comparison groups was the nature of the supplemental instruction each group received. Both groups received weekly 10- to 15-minute sessions of supplemental small-group instruction during circle time. The intervention group received supplemental instruction in numeracy: games they played with adults taught them to recognize numbers and count. The children first learned the numbers 1–10 and then focused on numbers 10–30. The comparison group participated in supplemental art activities during their sessions.

[20] There were two comparisons in **Kidd et al. (2008)**. In this comparison, the difference between the intervention and comparison groups was the nature of the supplemental instruction each group received. Both groups received weekly 10- to 15-minute sessions of supplemental small-group instruction during circle time. The intervention group received supplemental instruction in numeracy: games they played with adults taught them to recognize numbers and count. The children first learned the numbers 1–10 and then focused on numbers 10–30. The comparison condition participated in supplemental cognitive instruction: they played games to learn the oddity, seriation, and conservation.

[21] In **Lai, Baroody, and Johnson (2008)**, the difference between the intervention and comparison groups was the type of instruction in number and operations each group received. The intervention group participated in training sessions based on an adaptation of Gelman's magic task. Children practiced tasks that were addition only, subtraction only, and a mixture of addition and subtraction, over the course of the two-week training phase. The tasks helped to concretely teach reversibility (undoing operations). The comparison group played decomposition/composition games to help them estimate a large collection of 5 to 11 items.

[22] For Recommendation 1, the panel was not interested in comparisons between the three intervention conditions, although those findings are of interest in other recommendations.

[23] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups in teaching number and operations was whether they were taught number sense in a pull-out activity. The intervention group participated in a pull-out number sense curriculum using activities adapted from *Big Math for Little Kids*. The comparison group participated in pull-out sessions of the same length in which they read stories about shapes and patterns, rather than the number sense curriculum.

[24] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups in teaching number and operations was whether they were taught number sense in a pull-out activity. The intervention group participated in pull-out activities that involved reading stories to teach a number sense curriculum. The comparison group participated in pull-out sessions of the same length in which they read stories about shapes and patterns, rather than the number sense curriculum.

[25] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups in teaching number and operations was whether they were taught number sense in a pull-out activity. The intervention group participated in pull-out activities that involved movement to teach a number sense curriculum. The comparison group participated in pull-out sessions of the same length in which they read stories about shapes and patterns, rather than the number sense curriculum.

[26] Findings from these studies were previously reported in the WWC practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*. The panel reports the same findings as discussed in that practice guide.

[27] The effect is in the desired direction with the intervention making fewer errors than the comparison group, resulting in a negative effect size. However, to present the findings in a consistent manner, the effect size is reported as positive.

[28] In both **Ramani and Siegler (2008)** and **Siegler and Ramani (2008)**, the difference between the intervention and comparison groups in teaching number and operations was the nature of the board games played. The intervention group played a number-based version of *The Great Race*, with each space on the board having a number and children stating the number as they moved their token—thus practicing number and operations. The comparison group played *The Great Race* but with spaces that were colored, rather than numbered, and children stating the colors as they moved their token.

[29] In **Sophian (2004)**, the difference between the intervention and comparison groups was whether the children received math instruction using a researcher-developed, measurement-focused curriculum. The intervention group participated in a researcher-developed, measurement-focused curriculum that emphasized the concept of unit and taught number and operations. The comparison group participated in a literacy curriculum. There is no description of the math instruction children in the comparison group may have received as part of their regular classroom instruction.

Recommendation 2: Teach geometry, patterns, measurement, and data analysis using a developmental progression.

Level of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expertise and 12 randomized controlled trials[245] and 1 quasi-experimental study[246] that met WWC standards and examined interventions that addressed targeted instruction in one or more of the early math content areas of Recommendation 2 (see Table D.5). The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms. Positive effects were found in geometry, general numeracy, basic number concepts, operations, and patterns and classification;[247] however, some studies found no discernible effects.[248]

The panel believes that the most effective implementation of Recommendation 2 includes targeted instruction in geometry, patterns, measurement, and data analysis, using a developmental progression. The panel did not identify any evidence for the effects of teaching geometry, patterns, measurement, and data analysis based on any particular developmental progression. Such a study would have taught the same content to the intervention and comparison groups. The difference would have been the order in which the content was taught, with the intervention group receiving instruction based on a specific developmental progression and the comparison group receiving instruction in the same content in a different order. Based on their expertise and the positive effects found for interventions based on a developmental progression when compared to instruction that does not appear to be based on a developmental progression, the panel recommends the use of a developmental progression to guide instruction. Additional research is needed to identify the developmental progression that reflects how most children learn math.

Despite the presence of largely positive effects, when assigning the level of evidence for this recommendation, the panel identified three concerns regarding how well the evidence supported this recommendation: (1) interventions were multi-component, leading to concerns about how much of the demonstrated effect was the result of targeted teaching of geometry, patterns, measurement, and data analysis; (2) the degree to which the intervention and comparison groups received different amounts of targeted instruction in geometry, patterns, measurement, and data analysis could not be determined in all studies; and (3) many studies only reported on outcomes that were not aligned with the early math content areas included in this recommendation. As such, it was difficult for the panel to determine the extent to which targeted instruction in geometry, patterns, measurement, and data analysis according to a developmental progression was responsible for the effects seen in math achievement. Based on their expertise and the effects of interventions that include targeted instruction in geometry, patterns, measurement, and data analysis, the panel believes the studies generally support this recommendation, despite the limitations to the body of evidence.

Recommendation 2 addresses four distinct early math content areas: geometry, patterns, measurement, and data analysis. Interventions to teach math to young children frequently include more than one early math content area. In fact, 10 of the 13 studies included number and operations (Recommendation 1) in addition to at least one of the early math content areas discussed in Recommendation 2.[249]

- Teaching young children about geometry was included in 10 interventions[250] and examined in 12 studies.
- Patterns were a topic in 8 interventions[251] and examined in 10 studies.
- Measurement was a focus for 7 interventions[252] and examined in 10 studies.
- Data analysis was taught in 6 interventions[253] and examined in 8 studies.

The panel determined that, due to the multicomponent nature of many of the interventions comprising the body of evidence for Recommendation 2, it was not possible to attribute the demonstrated effects to the teaching of geometry, patterns, measurement, and data analysis.[254] In addition to teaching number and operations (Recommendation 1), many of the interventions examined also included aspects of Recommendations 3, 4, and 5. For example, *Building Blocks*, *EPIC*, and the *Pre-K Mathematics* curriculum include progress monitoring (the focus of Recommendation 3) as a core component of the intervention and involve the targeted teaching of number and operations (Recommendation 1), in addition to geometry, patterns, measurement, and data analysis. The panel cautions that the effects in these studies of comprehensive curricula may not be replicated when only the elements relating to Recommendation 2 are implemented. In fact, the panel believes that all the recommendations in this guide should be implemented together, as was the case in many of the interventions demonstrating positive effects.

When reviewing the evidence for Recommendation 2, the panel considered the degree to which the groups being compared differed (i.e., strength of the contrast) related to targeted instruction in geometry, patterns, measurement, and data analysis. The panel identified three studies in which the intervention group received targeted instruction, while the comparison group did not.[255] Although the specific nature of comparison group instruction was not clear in 6 of the 13 studies, the panel determined that the comparison group may have received targeted instruction in the same early math content areas.[256] This group of six studies found both positive[257] and no discernible[258] effects in the domains of operations, geometry, and general numeracy. Both the intervention and comparison groups received targeted instruction in the specific early math content areas in 4 of the 12 studies;[259] positive effects were found in these studies in the outcome domains of geometry,[260] general numeracy,[261] and basic number concepts.[262]

The panel identified a third concern regarding how well the evidence supported Recommendation 2: the match between the content of the intervention and the outcomes assessed was not exact. The studies reviewed used a variety of outcome measures, most of which were not specific to the early math content areas of geometry, patterns, measurement, and data analysis. For example, only outcomes in general numeracy—which may focus on number and operations—were reported in 6 of the 13 studies.[263] Mixed effects were found in the domains more closely related to the early math content areas that are the focus of Recommendation 2.[264]

The panel concluded that the body of evidence assessed in relation to Recommendation 2 was promising, but not aligned clearly enough with the panel's recommendation to support a moderate rating. The presence of aspects of all other recommendations made it difficult to determine whether the effects were due to targeted instruction in geometry, patterns, measurement, and data analysis. The panel was also concerned about the lack of specific information about how much time was spent on each early math content area in the intervention and comparison groups. Finally, many studies reported on outcomes that were not directly aligned with the early math content areas included in this recommendation. Together, these three limitations resulted in the panel not being able to claim with certainty that the effects seen were due solely to targeted instruction in the early math content areas of geometry, patterns, measurement, and data analysis.

Teaching geometry. All interventions examined included the deliberate teaching of geometry.[265] Positive effects were found for geometry, operations, and general numeracy outcomes, whether the teaching of shapes was part of a broader curriculum or the only component of the intervention. The interventions differed in whether they were identified as being based on a developmental progression overall (and presumably for teaching geometry)[266] or as providing activities for

teachers to use to teach geometry with no clear tie to a developmental progression.[267]

Building Blocks is a stand-alone math curriculum that includes whole-class as well as small-group activities and uses both physical and computer manipulatives. The curriculum focuses on two early math content areas: number (discussed in Recommendation 1) and geometry. The geometry portion of the *Building Blocks* curriculum includes activities for teaching shape identification, shape composition, congruence, construction of shapes, and turns.[268] In three studies, *Building Blocks* was the sole curriculum examined.[269] It was used in conjunction with the *Pre-K Mathematics* curriculum in one additional study.[270] *Pre-K Mathematics* and *DLM Early Childhood Express*, a curriculum that was developed by the *Building Blocks* developers and shares many key characteristics with *Building Blocks*, were examined in one study of preschool students.[271] Positive effects were found on children's performance in the geometry domain[272] and in the general numeracy domain[273] for children who participated in *Building Blocks* or *Pre-K Mathematics* with either *Building Blocks* or *DLM Early Childhood Express*, compared with children participating in their regular classroom instruction.

LOGO is a programming language that uses a turtle icon to follow directions and draw shapes or paths. Kindergarten children used *LOGO*—either the computer-based version or a floor-based version—to learn about paths and shapes.[274] Positive effects were seen in the geometry domain for children who used *LOGO*, compared with children who were not exposed to *LOGO*.

A comprehensive and integrated curriculum, *EPIC*, which includes early math content areas beyond number, was assessed in one study.[275] *EPIC* includes activities to teach children to recognize and identify critical attributes of shapes as well as combine shapes. Positive effects for the *EPIC* curriculum were reported in the domain of general numeracy, with Head Start children whose teachers used *EPIC* scoring higher on a general numeracy outcome than their comparison counterparts whose teachers used *DLM Early Childhood Express* with the *High/Scope Educational Research Foundation's Preschool Child Observation Record*—another comprehensive curriculum that includes math.

Three researcher-developed curricula included a focus on geometry. One curriculum focused on the discovery of relations between shapes, including understanding part-whole relations and using those relations to compare areas. A positive effect was found for preschool children's scores in the domain of a general numeracy.[276] In a second study, an equivalent number of unit blocks of each shape and size were provided to the groups for use during block-building sessions.[277] Children in intervention groups with specific block-building instruction were given sequenced instruction and block-building activities, including problems to solve. The comparison group participated in regular classroom math instruction and was given additional unstructured small-group time for block-building sessions. Kindergartners who received block-building instruction generally scored higher on shape and geometry outcomes than the comparison children in the regular classroom instruction (who did not receive the specific block-building instruction).[278]

Teaching patterns. Interventions including a focus on patterns were assessed in 10 studies.[279] Both *Building Blocks* and *Pre-K Mathematics* include units focusing on identifying and extending patterns. *LOGO*, which teaches children about shapes, paths, and distance, was examined in the other study that included a focus on patterns. All studies reported positive or no discernible effects in the domains of general numeracy, basic number concepts, operations, geometry, and patterns and classification.[280] One study included a pattern-specific outcome; the study found that children who participated in *Building Blocks* scored higher on the pattern subscale of the REMA than children participating in regular classroom instruction, including children who received instruction

using the *DLM Early Childhood Express* curriculum.[281] *Building Blocks* or *DLM Early Childhood Express*, either with or without the *Pre-K Mathematics* curriculum, was the tested intervention in five of the studies.[282] Seven of the interventions discussed in these studies[283] were also examined as evidence for the effects of teaching geometry to children, as previously discussed.

Teaching measurement. Curricula that focused on teaching children measurement were assessed in 10 studies implementing 7 interventions.[284] The interventions included lessons to support teaching children to make comparisons in length, weight, and capacity.[285] Additionally, three of the interventions supported the use of nonstandard and standard measurement tools.[286] All of the studies reviewed had positive or no discernible effects. Positive effects were found in the domains of general numeracy, geometry, and basic number concepts when intervention children were compared with children who did not receive targeted instruction in measurement.[287] *Building Blocks* was the focal curriculum in 3 of the 10 studies[288] and was combined with the *Pre-K Mathematics* curriculum in an additional study.[289] The *Pre-K Mathematics* curriculum in combination with *DLM Early Childhood Express* was examined in one study.[290] Another study focused on *LOGO* and its effects on the geometry scores of children.[291] *LOGO* supports children in working with a coordinate axis, entering coordinates to assess length, and comparing the resulting shapes. Studies of *Creative Curriculum* and *Bright Beginnings* found no discernible effects in the general numeracy, operations, and geometry domains.[292] A researcher-developed curriculum that focused on measurement was developed to teach children the quantitative aspects of units. Although measurement was not a concrete or explicit component of the curriculum, lessons familiarized children with units of quantification and supported learning about measurement or quantitative comparisons.[293]

Teaching data analysis. Five studies, all of which included *Building Blocks* or *DLM Early Childhood Express* as the intervention, examined the effects of deliberately teaching data analysis (including graphing).[294] *Building Blocks* includes activities to teach children to classify, represent, and use information to ask and answer questions. These activities could include graphing and other forms of data analysis. Compared with children who did not receive targeted instruction in an element of data analysis, children who participated in *Building Blocks* or the *Pre-K Mathematics* curriculum combined with either *Building Blocks* or *DLM Early Childhood Express* scored higher on assessments in the domains of general numeracy and basic number concepts. No negative effects were reported. Two additional studies evaluating curricula that included data analysis activities reported no discernible effects in the domains of general numeracy, operations, and geometry.[295] The final study, evaluating *EPIC*, found positive effects on an outcome in the general numeracy domain.[296]

Table D.5. Studies of interventions that taught geometry, patterns, measurement, or data analysis and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested				
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Geometry**	**Teach Patterns**	**Teach Measurement**	**Teach Data Analysis**	**Use a Developmental Progression**
Barnett et al. (2008) RCT Meets evidence standards with reservations	Children attending a full-day preschool program Children: 202 total (85 intervention; 117 comparison) Age range: 3 to 4 years; slightly more 4-year-olds (54%)	*Tools of the Mind* vs. regular classroom instruction (district-created, balanced literacy)	Operations: WJ-Revised–Applied Math Problems Subtest No discernible (0.17, ns)	X[5]	X[5]			X[5]
Casey et al. (2008)[6,7] RCT Meets evidence standards without reservations	Six kindergarten classrooms in two urban school districts Children: 71 total (35 intervention; 36 comparison) Age range: 5.6 to 6.7 years	Sequenced block-building activities in storytelling context vs. regular classroom instruction	Geometry: Building Block Test Positive (0.52, ns)	X[8]				X[8]
			Geometry: WISC-IV Block Design Subtest Positive (0.43, ns)	X[8]				X[8]
			Geometry: Mental Rotation No discernible (–0.22, ns)	X[8]				X[8]
Casey et al. (2008)[6,7] RCT Meets evidence standards without reservations	Six kindergarten classrooms in two urban school districts Children: 65 total (29 intervention; 36 comparison) Age range: 5.6 to 6.7 years	Sequenced block-building activities vs. regular classroom instruction	Geometry: Building Block Test No discernible (0.06, ns)	X[9]				X[8]
			Geometry: WISC-IV–Block Design Subtest Positive (0.35, ns)	X[9]				X[8]
			Geometry: Mental Rotation No discernible (0.16, ns)	X[9]				X[8]
Clements and Sarama (2007b)[6,10] RCT Meets evidence standards with reservations	Preschool classrooms in state-funded or Head Start programs Children: 68 total (30 intervention; 38 comparison) Age range: 2.9 to 4.8 years Mean age: 4.2 years (SD 6.2 months)	*Building Blocks* vs. regular classroom instruction (*Creative Curriculum* or locally developed)	Basic number concepts: BB Assessment–Number Scale Positive (0.75*)	X[11]	X[11]	X[11]	X[11]	X[11]
			Geometry: BB Assessment–Geometry Scale Positive (1.40*)	X[11]	X[11]	X[11]	X[11]	X[11]

(continued)

Table D.5. Studies of interventions that taught geometry, patterns, measurement, or data analysis and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested				
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Geometry**	**Teach Patterns**	**Teach Measurement**	**Teach Data Analysis**	**Use a Developmental Progression**
Clements and Sarama (2008)[6,12] RCT Meets evidence standards without reservations	24 teachers in Head Start or state-funded preschool programs were randomly assigned to one of three conditions. 20 teachers in programs serving low- and middle-income students were randomly assigned to one of two conditions. Children: 201 total (101 intervention; 100 comparison) Children had to be within kindergarten entry range for the following year.	*Building Blocks* vs. regular classroom instruction (locally developed)	General numeracy: REMA Positive (1.07*)	?	?	?	?	?
Clements et al. (2011)[6,7,13] RCT Meets evidence standards without reservations	Prekindergarten classrooms in two urban public school districts Children: 1,305 total (927 intervention; 378 comparison)	*Building Blocks* vs. regular classroom instruction (*Where Bright Futures Begin*; *Opening the World of Learning*; *Investigations in Number, Data, and Space*; *DLM Early Childhood Express*)	General numeracy: REMA–Total Positive (0.48*)	X[14]	X[14]	X[14]	X[14]	X[14]
			Basic number concepts: REMA–Numbers Total Positive (0.39*)	X[14]	X[14]	X[14]	X[14]	X[14]
			Geometry: REMA–Geometry Total Positive (0.64*)	X[14]	X[14]	X[14]	X[14]	X[14]
Fantuzzo, Gadsden, and McDermott (2011)[15] RCT Meets evidence standards without reservations	80 Head Start classrooms in Philadelphia, Pennsylvania Children: 778 total (397 intervention; 381 comparison) Age range: 2.9 to 5.8 years Mean age: 4.2 years (SD 6.8 months)	*Evidence-based Program for Integrated Curricula (EPIC)* vs. regular classroom instruction (*DLM Early Childhood Express*)	General numeracy: LE–Mathematics, Wave 4 Positive (0.18*)	X[16]		X[16]	X[16]	X[16]

(continued)

Table D.5. Studies of interventions that taught geometry, patterns, measurement, or data analysis and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested				
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Geometry**	**Teach Patterns**	**Teach Measurement**	**Teach Data Analysis**	**Use a Developmental Progression**
Kidd et al. (2008)[7] RCT Meets evidence standards without reservations	Children attending kindergarten in a metropolitan school district Children: 52 total (26 intervention; 26 comparison) Age: All were 5 years old by the end of September the year the intervention was implemented.	cognitive instruction in oddity, seriation, and conservation vs. treated comparison (art)	Basic number concepts: Conservation Test Positive (0.49, ns)		X[17]			
			Operations: WJ-III Applied Problems Positive (0.71*)		X[17]			
			Patterns and classification: Oddity Test Positive (0.87*)		X[17]			
			Patterns and classification: OLSAT Classification Scale Positive (0.45, ns)		X[17]			
			Patterns and classification: Seriation Test Positive (0.62*)		X[17]			
Kidd et al. (2008)[7] RCT Meets evidence standards without reservations	Children attending kindergarten in a metropolitan school district Children: 52 total (26 intervention; 26 comparison) Age: All were 5 years old by the end of September the year the intervention was implemented.	cognitive instruction in oddity, seriation, and conservation vs. treated comparison (numeracy)	Basic number concepts: Conservation Test Positive (0.68*)		X[18]			
			Operations: WJ-III Applied Problems Positive (0.50, ns)		X[18]			
			Patterns and classification: Oddity Test Positive (0.68*)		X[18]			
			Patterns and classification: OLSAT Classification Scale Positive (0.46, ns)		X[18]			
			Patterns and classification: Seriation Test Positive (0.64, ns)		X[18]			
Klein et al. (2008)[6,7] RCT Meets evidence standards without reservations	40 prekindergarten classrooms in Head Start or state-funded programs in New York and California Children: 278 total (138 intervention; 140 comparison) Age range: 3.8 to 4.9 years Mean age: 4.4 years	*Pre-K Mathematics* combined with *DLM Early Childhood Express* vs. regular classroom instruction (*Creative Curriculum*, *High Scope*, Montessori, locally developed)	General numeracy: CMA Positive (0.57*)	X[19]	X[19]	X[19]	X[19]	X[19]

(continued)

Table D.5. Studies of interventions that taught geometry, patterns, measurement, or data analysis and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested				
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Geometry**	**Teach Patterns**	**Teach Measurement**	**Teach Data Analysis**	**Use a Developmental Progression**
PCER Consortium (2008, Chapter 2)[6,20] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 193 total (93 intervention; 100 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.17, ns)	?	?	?	?	?
			General numeracy: CMA-A, Posttest No discernible (0.10, ns)	?	?	?	?	?
			Geometry: Shape Composition, Posttest No discernible (–0.12, ns)	?	?	?	?	?
PCER Consortium (2008, Chapter 2)[6,20] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 198 total (98 intervention; 100 comparison) Mean age: 4.5 years	*Bright Beginnings* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.16, ns)	?	?	?	?	
			General numeracy: CMA-A, Posttest No discernible (0.14, ns)	?	?	?	?	
			Geometry: Shape Composition, Posttest No discernible (–0.03, ns)	?	?	?	?	
PCER Consortium (2008, Chapter 3)[21] RCT Meets evidence standards with reservations	Preschoolers attending Head Start centers Children: 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.20, ns)	?	?	?	?	?
			General numeracy: CMA-A–Mathematics Composite, Posttest No discernible (–0.10, ns)	?	?	?	?	?
			Geometry: Shape Composition, Posttest No discernible (0.19, ns)	?	?	?	?	?
			Operations: WJ-III–Applied Problems, Maintenance (spring of kindergarten year) No discernible (0.09, ns)	?	?	?	?	?

(continued)

Table D.5. Studies of interventions that taught geometry, patterns, measurement, or data analysis and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested				
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Teach Geometry**	**Teach Patterns**	**Teach Measurement**	**Teach Data Analysis**	**Use a Developmental Progression**
PCER Consortium (2008, Chapter 3)[21] RCT Meets evidence standards with reservations *(continued)*	Preschoolers attending Head Start centers Children: 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	General numeracy: CMA-A–Mathematics Composite, Maintenance (spring of kindergarten year) No discernible (0.14, ns)	?	?	?	?	?
			Geometry: Shape Composition, Maintenance (spring of kindergarten year) No discernible (–0.01, ns)	?	?	?	?	?
Sarama et al. (2008)[22] RCT Meets evidence standards without reservations	Head Start or state-funded prekindergarten classrooms in New York and California Children: 200 total (104 intervention; 96 comparison) Average age: 4.3 years	*Building Blocks* combined with *Pre-K Mathematics* vs. regular classroom instruction	General numeracy: REMA Positive (0.62*)	?	?	?	?	?
Sophian (2004)[6,7] QED Meets evidence standards with reservations	Head Start sites Children: 94 total (46 intervention; 48 comparison) Age range: 2 years, 6 months to 4 years, 7 months	Researcher-developed measurement-focused curriculum vs. treated comparison (literacy instruction)	General numeracy: DSC–Mathematics Subscale Positive (0.33, ns)	X[23]		X[23]		
Weaver (1991)[24] RCT Meets evidence standards without reservations	Kindergartners in a middle-class suburban elementary school Children: 79 total (39 intervention; 40 comparison) Age range: 4 to 5 years	*LOGO* (floor and screen) vs. treated comparison (*Path* instruction and regular classroom instruction)	Geometry: Geometry Score Positive (0.86*)	X[25]	X[25]	X[25]		

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that taught geometry, patterns, measurement and data analysis, and that may have used a developmental progression to guide that instruction.

X The intervention included this component.

BB Assessment = Building Blocks Assessment of Early Mathematics[297]
CMA = Child Math Assessment[298]
CMA-A = Child Math Assessment–Abbreviated[299]
DSC = Developing Skills Checklist[300]
LE = Learning Express[301]
OLSAT = Otis-Lennon School Ability Test[302]
REMA = Research-Based Early Math Assessment[303]
WISC-IV = Wechsler Intelligence Scale for Children, fourth edition[304]
WJ-Revised = Woodcock-Johnson, revised edition[305]
WJ-III = Woodcock-Johnson, third edition[306]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" refers to findings that are neither significant nor substantively important.

[5] In **Barnett et al. (2008)**, the difference between the intervention and comparison groups with respect to math instruction is not known. The intervention group participated in *Tools of the Mind*, a comprehensive early childhood curriculum with a math component that supported incorporating math into other parts of the school day. The comparison group participated in a district-created balanced literacy curriculum. From the information provided, it was not clear how the intervention and comparison groups differed with respect to instruction in the early math content areas of geometry and patterns or the use of a developmental progression to guide instruction in these early math content areas.

[6] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[7] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[8] In **Casey (2008)**, the difference between the intervention and comparison groups in teaching measurement was in the instructional use of building blocks. The intervention group engaged in sequenced block-building activities in a story-telling context. The comparison group had the opportunity to engage in unstructured play with blocks. There is not sufficient information to know what other activities were conducted in to teach geometry.

[9] In **Casey (2008)**, the difference between the intervention and comparison groups in teaching measurement was in the instructional use of building blocks. The intervention group engaged in sequenced block-building activities. The comparison group had regular math instruction. There is not sufficient information to know what other activities were conducted in the comparison group to teach geometry.

[10] **Clements and Sarama (2007b)** also reported scores for the subscales of the Number and Geometry scales; positive effects were seen for each subscale. Findings from **Clements and Sarama (2007b)** were previously reported in the WWC intervention report on *SRA Real Math Building Blocks PreK*. The panel reports the same findings as presented in the intervention report.

[11] In **Clements and Sarama (2007b)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the curricula used in the comparison classrooms, including the branded comprehensive early childhood curriculum *Creative Curriculum*. The intervention group participated in *Building Blocks*, a math curriculum that included instruction in geometry, patterns, measurement, and data analysis guided by a developmental progression. The comparison group participated in a variety of curricula, including *Creative Curriculum*, which also included instruction in geometry, patterns, measurement, and data analysis guided by a developmental progression.

[12] For **Clements and Sarama (2008)**, the WWC is reporting author-reported effect sizes consistent with prior reporting of findings from this study in the WWC intervention report on *SRA Real Math Building Blocks PreK*.

[13] **Clements et al. (2011)** also reported the subscale scores from the REMA. Findings for the subscale scores were consistent with the total score findings and generally positive (9 of 13 scores). No discernible effects were seen for 4 of the 13 subscale scores (two in the geometry domain: transformations/turns and comparing shapes; one in the operations domain: arithmetic; and one in the basic number concepts domain: composition of number).

[14] In **Clements et al. (2011)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the various branded curricula used in the comparison classrooms, including *DLM Early Childhood Express*, a comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that included instruction in geometry, patterns, measurement, and data analysis guided by a developmental progression. The comparison group participated in a number of branded curricula, including *DLM Early Childhood Express*, an early childhood curriculum that included instruction in geometry, patterns, measurement, and data analysis but was not guided by a developmental progression in the same manner as *Building Blocks* instruction.

[15] **Fantuzzo, Gadsden, and McDermott (2011)** reported on four waves of data collection. The panel decided to use Wave 1 as pretest data, because it was collected prior to the delivery of math content. Wave 4 was used as the posttest, as it was collected

at the end of the school year and delivery of the intervention. Waves 2 and 3 could be viewed as intermediary outcomes, but the panel chose to focus on posttests when determining levels of evidence.

[16] In **Fantuzzo, Gadsden, and McDermott (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *EPIC* and *DLM Early Childhood Express*, a branded comprehensive early childhood curriculum. The intervention group participated in *EPIC*, a comprehensive early childhood curriculum that included instruction in geometry, measurement, and data analysis guided by a developmental progression. The comparison group participated in another branded comprehensive early childhood curriculum, *DLM Early Childhood Express*, which included math content in the early math content areas of geometry, patterns, measurement, and data analysis but was not guided by a developmental progression in the same manner as instruction using *EPIC*.

[17] There were two comparisons in **Kidd et al. (2008)**. In this comparison, the difference between the intervention and comparison groups was the nature of the supplemental instruction each group received. Both groups received weekly 10- to 15-minute sessions of supplemental small-group instruction during circle time. The intervention group received supplemental cognitive instruction in oddity, seriation, and conservation: games they played with adults enabled them to practice these concepts. The comparison group participated in supplemental art activities during their sessions.

[18] There were two comparisons in **Kidd et al. (2008)**. In this comparison, the difference between the intervention and comparison groups was the nature of the supplemental instruction each group received. Both groups received weekly 10- to 15-minute sessions of supplemental small-group instruction during circle time. The intervention group received supplemental cognitive instruction in oddity, seriation, and conservation; games they played with adults enabled them to practice these concepts. The comparison group participated in supplemental instruction in numeracy–they played games with adults that taught them to recognize numbers and count. The children first learned the numbers 1–10 and then focused on numbers 10–30.

[19] In **Klein et al. (2008)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between the combined *Pre-K Mathematics* curriculum and *DLM Early Childhood Express* intervention and the curricula used in the comparison classrooms, including *Creative Curriculum*. The intervention group, which participated in a combination of *Pre-K Mathematics* and *DLM Early Childhood Express*, received instruction in geometry, patterns, measurement, and data analysis using a developmental progression. The comparison group participated in a number of branded curricula, including *Creative Curriculum*, a comprehensive early childhood curriculum that included instruction in geometry, patterns, measurement, and data analysis guided by a developmental progression.

[20] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel rated the study differently but reports the same findings as presented in the intervention report. The difference in study rating is due to the use of WWC Version 2.1 standards as opposed to WWC Version 1.0 standards. Findings from this study of *Bright Beginnings* were previously reported in the WWC intervention report on *Bright Beginnings*. The panel reports the same findings as presented in the intervention report. For both *Creative Curriculum* and *Bright Beginnings*, the authors report on additional outcomes that were assessed in the spring of kindergarten.

[21] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel reports the same findings as presented in the intervention report.

[22] **Sarama et al. (2008)** reported subscale scores as well; however, only the means were provided, so the WWC was unable to calculate effect sizes for the subscales.

[23] In **Sophian (2004)**, the difference between the intervention and comparison groups was whether children received math instruction using a researcher-developed, measurement-focused curriculum. The intervention group participated in a researcher-developed, measurement-focused curriculum that emphasized the concept of unit and taught geometry and measurement. The comparison group participated in a literacy curriculum. There was no description of the math instruction children in the comparison group may have received as part of their regular classroom instruction.

[24] **Weaver (1991)** also assessed the impact of computer-based *LOGO* compared with floor-based *LOGO* for preschool children. This contrast is not evidence for this recommendation, as both groups of children used *LOGO*; there were no discernible effects found for computer-based *LOGO*.

[25] In **Weaver (1991)**, the difference between the intervention and comparison groups was the use of a *LOGO* turtle (floor- or computer-based) to practice geometry concepts including paths, knowledge of right and left, and perspective-taking abilities. The intervention group and a portion of the comparison group (the "path" portion) participated in three non-computer lessons on the properties of paths. The other portion of the comparison group, the control group, did not receive any portion of the "path" curriculum. The intervention group then worked with either a floor- or computer-based turtle to practice concepts related to paths. *LOGO* is a programming language that results in a turtle drawing a path while following commands entered by the child to indicate both a graphic command (right, left, forward, or backward) and a distance to move or the degrees to turn.

Recommendation 3: Use progress monitoring to ensure that math instruction builds on what each child knows.

Level of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expertise and 11 randomized controlled trials[307] and 1 quasi-experimental study[308] that met WWC standards and examined interventions that included at least one component of Recommendation 3 (see Table D.6). The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms. Nine studies found positive effects for general numeracy, operations, basic number concepts, and geometry.[309] One study found positive effects in general numeracy and both positive and no discernible effects in the domains of operations.[310] Two studies found no discernible effects in the general numeracy, operations, and geometry domains.[311]

The panel believes that the most effective implementation of Recommendation 3 requires deliberate and consistent implementation of a progress-monitoring approach that establishes a child's level of knowledge, tailors instruction to a child's individual needs and developmental level, and monitors progress to facilitate the child's building connections between new math knowledge and what the child already knows. The 12 studies examined interventions that included key components of other recommendations as well, making it difficult to attribute the demonstrated effects to the activities related to using progress monitoring to ensure that math instruction builds on what each child knows. Additionally, the panel cautions that the difference in the experiences of the intervention and comparison groups with respect to the use of progress monitoring may not be large enough to consider the study a direct test of Recommendation 3, as the comparison group, in six cases, received progress monitoring, too.[312] Based on their expertise and the effects of interventions that include progress monitoring, the panel believes the studies generally support this recommendation despite the limitations to the body of evidence.

The 12 studies reviewed included interventions that provided targeted instruction in number and operations, geometry, patterns, measurement, and data analysis to varying degrees (Recommendations 1 and 2). Similarly, all 12 studies were identified as providing opportunities to support children in viewing and describing their world mathematically (Recommendation 4). Additionally, the panel found that all 12 studies examined interventions that, in addition to testing key components of Recommendation 3, included key components of Recommendation 5 (teaching math through dedicated sessions and integrating math into other aspects of the school day). The acknowledged overlap illustrates the panel's belief that early math instruction should incorporate the key components of all five recommendations when possible.

The panel examined the extent to which instruction in the intervention and comparisons groups differed with respect to the use of progress monitoring and building upon the child's existing knowledge. The panel identified two studies in which the intervention group participated in additional math instruction with progress monitoring, while the comparison group did not; positive and no discernible effects were found.[313] In another four studies, the comparison group may have received math instruction that included progress monitoring as envisioned by the panel.[314] Positive effects in the general numeracy outcome domain were seen in two of these four studies.[315] No discernible effects in the general numeracy, operations, and geometry outcome domains were seen in the other two studies.[316] Two of the 12 studies did not include progress monitoring, but the intervention emphasized starting with a child's informal knowledge; positive effects in the outcome domains of general numeracy[317] and basic number concepts[318] were seen in these two studies. Four of the 12 studies focused on interventions that included both progress

monitoring and emphasis on starting with a child's informal knowledge; however, the comparison group was identified as participating in an intervention that included at least one of these two components as well.[319] Positive effects were reported in outcome domains of general numeracy,[320] basic number concepts,[321] and geometry.[322]

The panel did not view the evidence as sufficient to warrant a moderate evidence rating for two key reasons. First, the studies incorporated practices associated with Recommendation 3 and practices associated with the other recommendations in the guide (i.e., they were multi-component interventions).[323] For example, all studies included targeted instruction in number and operations (see Recommendation 1), and 8 of the 12 included targeted instruction in at least one of the early math content areas addressed in Recommendation 2.[324] As such, it was difficult for the panel to determine the extent to which the use of progress monitoring was responsible for the effects seen in math achievement. Second, the difference in the amount and type of progress monitoring the intervention and comparison groups received was not distinct in most studies[325] and thus was not a direct test of a key component of the recommendation. The panel believes progress monitoring should be a deliberate process that identifies a child's knowledge as a starting point and regularly assesses progress in developing connections between new math knowledge and what the child already knows. The panel further believes that when progress monitoring is implemented with other recommendations in this guide, it will lead to improved math achievement for children.

Progress monitoring to tailor instruction. Four studies examined the *Building Blocks* curriculum and found consistently positive effects of the intervention on math outcomes.[326] *Building Blocks* incorporates assessments, including informal assessments such as small-group recording sheets and software, as a key component of learning and instruction. Teacher training in the *Building Blocks* curriculum focuses on adapting activities and tailoring instruction based on teachers' knowledge of the children's math abilities.

Two studies[327] examining the *Building Blocks* curriculum were conducted in low-income, urban prekindergarten classrooms. On average, children receiving the intervention in the first study [328] performed better on assessments in geometry and basic number concepts than the children in the comparison group, who received the regular classroom math curriculum. In the second study, the authors also found positive effects of the *Building Blocks* curriculum on student achievement on a general numeracy assessment.[329] The same researchers conducted a third study[330] of the *Building Blocks* curriculum among children in Head Start and state-funded prekindergarten programs serving low-income and mixed-income populations. The study found that, on average, the children receiving the *Building Blocks* curriculum outperformed the comparison group on the Research-Based Early Math Assessment (REMA).[331]

The fourth study[332] of *Building Blocks* implemented an intervention in New York and California Head Start programs and state-funded prekindergarten classrooms. The intervention in this study combined the software component of the curriculum and correlated non-computer activities with twice-weekly small-group sessions from the *Pre-K Mathematics* curriculum. The combined program was designed to follow research-based learning trajectories specifying developmental progressions of levels of competence. The children receiving the intervention curriculum performed significantly better, on average, than their counterparts (who received the regular classroom math curriculum) on the REMA, a measure of general numeracy.

The panel identified six additional studies examining interventions that combined regular math lessons with informal and formal assessments to monitor progress and tailor instruction. The first study,[333] conducted in 40 Head Start and state-funded prekindergarten programs in New York and California,

tested the effectiveness of a math intervention that combined elements from the *Pre-K Mathematics* curriculum and *DLM Early Childhood Express*. The curriculum provided teachers with suggestions for scaffolding activities and downward and upward extensions of the activities. Assessment record sheets for the small-group activities allowed teachers to track the progress of individual children, and time was built into the curriculum for reviewing activities with children experiencing difficulty. Children in the comparison group continued to receive regular classroom instruction (the preschool curricula used in their programs). The study found that, on average, the intervention had a positive effect on children's general numeracy as measured by the Child Math Assessment (CMA).[334]

Researchers separately examined two interventions, *Bright Beginnings* and *Creative Curriculum*, in a second study conducted in prekindergarten classrooms in Tennessee.[335] *Creative Curriculum* was also examined in Head Start centers.[336] Both curricula included a dedicated math component as part of a larger comprehensive curriculum and used assessments for ongoing progress monitoring. Children in the comparison group with regular classroom instruction were taught using teacher-developed curricula that focused on school readiness. The authors made two comparisons: (1) between the *Bright Beginnings* group and the regular classroom instruction comparison group, and (2) between the *Creative Curriculum* group and the regular classroom instruction comparison group. They found no discernible effects of either intervention on prekindergartners' performance on assessments of general numeracy, operations, and geometry. Further, no discernible effects were found for *Creative Curriculum* in the Head Start centers.

Although the interventions examined in three other studies[337] included progress monitoring, the contrast between the intervention and regular classroom instruction comparison conditions in these studies was not as strong as those drawn in the studies described above. In the three studies, children in the comparison group received instruction in curricula that used regular assessments for progress monitoring. As such, the positive effects of these interventions on children's math performance may not have been due solely to the use of progress monitoring. The curricula tested in the studies also included components such as additional scripted time spent on developing number sense, which may have contributed to the positive effects found by the authors.

In two of these studies, researchers administered a supplemental number sense curriculum to low-income kindergarten children.[338] The intervention was administered in small groups that met three times weekly to participate in carefully scripted activities on topics including number recognition, counting, basic addition and subtraction, and magnitude. The education students who implemented the intervention used informal assessments to monitor children's progress and provide individually tailored review material. Both the intervention and comparison children also continued to receive the regular classroom math instruction from the stand-alone *Math Trailblazers* or *Math Connects* curricula. *Math Trailblazers* incorporated a variety of informal and formal assessments, which informed teachers about children's learning and could then be used to guide instruction. The authors found positive effects of the supplemental curriculum on operations and general numeracy measures administered to the children both immediately following the intervention (posttest) and at a later maintenance assessment (6 weeks).

The third study focused on the *Evidence-based Program for Integrated Curricula (EPIC)*, a comprehensive, stand-alone curriculum that was not specific to math.[339] Teachers used curriculum-based assessments, called *EPIC* Integrated Check-Ins (ICIs), three times yearly to identify competencies of individual children, monitor progress, and inform instruction. Researchers randomly assigned 80 Head Start classrooms to implement either *EPIC* or

DLM Early Childhood Express, another standalone curriculum. Teachers implementing the comparison curriculum, *DLM Early Childhood Express*, used the *High/Scope Educational Research Foundation's Preschool Child Observation Record* to conduct individual assessments of children and monitor their progress. The researchers noted that teachers in both the intervention and comparison conditions conducted a comparable number of assessments. On an assessment of general numeracy, children taught using *EPIC* performed better, on average, than students whose teachers used *DLM Early Childhood Express*.

There are two studies included in the body of evidence for this recommendation that emphasized building on children's informal knowledge, but did not include a deliberate progress monitoring process.[340] The first study used the *Math Is Everywhere* curriculum, which provided teachers with the supports necessary to teach math through small groups, as well as at transitions and meal times.[341] Teachers were given 85 activities that could be modified to fit teaching styles and children's interests and abilities. Children in *Math Is Everywhere* classrooms had, on average, higher scores on a measure of general numeracy than children whose teachers did not receive the activities to support building on preexisting knowledge. The second study used the *Rightstart* curriculum, which involved 20 lessons that taught number sense based on three instructional principles: activities to facilitate making connections, exploration and discussion of concepts, and using current understanding to construct new understanding at the next level.[342] Children participating in *Rightstart* classrooms scored, on average, higher on a measure of basic number concepts than children in regular classrooms.

Table D.6. Studies of interventions that used a deliberate progress-monitoring process and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Use Progress Monitoring to Tailor Instruction**	**Start with a Child's Informal Math Knowledge**
Arnold et al. (2002)[5] RCT Meets evidence standards without reservations	Pairs of half-day or full-day Head Start classes Children: 103 total (49 intervention; 54 comparison) Age range: 3.1 to 5.3 years Average age: 4.4 years (SD 7.32 months)	*Math Is Everywhere* vs. regular classroom instruction	General numeracy: TEMA-2 Positive (0.40, ns)		?
Clements and Sarama (2007b)[5,6] RCT Meets evidence standards with reservations	Preschool classrooms in state-funded or Head Start programs Children: 68 total (30 intervention; 38 comparison) Age range: 2.9 to 4.8 years Mean age: 4.2 years (SD 6.2 months)	*Building Blocks* vs. regular classroom instruction (*Creative Curriculum*, locally developed)	Basic number concepts: BB Assessment–Number Scale Positive (0.75*)	X[7]	X[7]
			Geometry: BB Assessment–Geometry Scale Positive (1.40*)	X[7]	X[7]

(continued)

Table D.6. Studies of interventions that used a deliberate progress-monitoring process and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Use Progress Monitoring to Tailor Instruction**	**Start with a Child's Informal Math Knowledge**
Clements and Sarama (2008)[5,8] RCT Meets evidence standards without reservations	24 teachers in Head Start or state-funded preschool programs were randomly assigned to one of three conditions. 20 teachers in programs serving low- and middle-income students were randomly assigned to one of two conditions. Children: 201 total (101 intervention; 100 comparison) Children had to be within kindergarten entry range for the following year.	*Building Blocks* vs. regular classroom instruction (locally developed)	General numeracy: REMA Positive (1.07*)	?	?
Clements et al. (2011)[5,9,10] RCT Meets evidence standards without reservations	Prekindergarten classrooms in two urban public school districts Children: 1,305 total (927 intervention; 378 comparison)	*Building Blocks* vs. regular classroom instruction (*Where Bright Futures Begin*; *Opening the World of Learning*; *Investigations in Number, Data, and Space*; *DLM Early Childhood Express*)	General numeracy: REMA–Total Positive (0.48*)	X[11]	X[11]
			Basic number concepts: REMA–Numbers Total Positive (0.39*)	X[11]	X[11]
			Geometry: REMA–Geometry Total Positive (0.64*)	X[11]	X[11]
Dyson, Jordan, and Glutting (2013)[12] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 121 total (56 intervention; 65 comparison) Mean age: 5.5 years (SD 4.0 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers*)	General numeracy: NSB–Total Score, Posttest Positive (0.64*)	X[13]	
			Operations: WJ-III–Total Score, Posttest Positive (0.29, ns)	X[13]	
			General numeracy: NSB–Total Score, Maintenance (6 weeks) Positive (0.65*)	X[13]	
			Operations: WJ-III–Total Score, Maintenance (6 weeks) No discernible (0.18, ns)	X[13]	
Fantuzzo, Gadsden, and McDermott (2011)[14] RCT Meets evidence standards without reservations	80 Head Start classrooms in Philadelphia, Pennsylvania Children: 778 total (397 intervention; 381 comparison) Age range: 2.9 to 5.8 years Mean age: 4.2 years (SD 6.8 months)	*Evidence-based Program for Integrated Curricula (EPIC)* vs. regular classroom instruction (*DLM Early Childhood Express*)	General numeracy: LE–Mathematics, Wave 4 Positive (0.18*)	X[15]	X[15]

(continued)

Table D.6. Studies of interventions that used a deliberate progress-monitoring process and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Use Progress Monitoring to Tailor Instruction**	**Start with a Child's Informal Math Knowledge**
Griffin, Case, and Capodilupo (1995) and related publication Griffin, Case, and Siegler (1994)[16] QED Meets evidence standards with reservations	Kindergarten students in public schools in inner-city areas in Massachusetts Children: 47 total (23 intervention; 24 comparison)	*Rightstart* vs. regular classroom instruction	Basic number concept: NKT Positive (1.79*)		?
Jordan et al. (2012)[17] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 86 total (42 intervention; 44 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers* or *Math Connects*)	General numeracy: NSB–Total, Posttest Positive (1.10*)	X[18]	
			Operations: WJ-III–Total, Posttest Positive (0.91*)	X[18]	
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.77*)	X[18]	
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.56*)	X[18]	
Jordan et al. (2012)[17] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 84 total (42 intervention; 42 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. treated comparison (supplemental language intervention with *Math Trailblazers* or *Math Connects*)	General numeracy: NSB, Total, Posttest Positive (0.91*)	X[19]	
			Operations: WJ-III, Total, Posttest Positive (0.84*)	X[19]	
			General numeracy: NSB, Total, Maintenance (8 weeks) Positive (0.62*)	X[19]	
			Operations: WJ-III, Total, Maintenance (8 weeks) Positive (0.75*)	X[19]	
Klein et al. (2008)[5] RCT Meets evidence standards without reservations	40 prekindergarten classrooms in Head Start or state-funded programs in New York and California Children: 278 total (138 intervention; 140 comparison) Age range: 3.8 to 4.9 years Mean age: 4.4 years	*Pre-K Mathematics* combined with *DLM Early Childhood Express* vs. regular classroom instruction (*Creative Curriculum*, *High Scope*, Montessori, locally developed)	General numeracy: CMA Positive (0.57*)	X[20]	X[20]

(continued)

Table D.6. Studies of interventions that used a deliberate progress-monitoring process and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Use Progress Monitoring to Tailor Instruction**	**Start with a Child's Informal Math Knowledge**
PCER Consortium (2008, Chapter 2)[5,21] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 193 total (93 intervention; 100 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.17, ns)	?	
			General numeracy: CMA-A, Posttest No discernible (0.10, ns)	?	
			Geometry: Shape Composition, Posttest No discernible (–0.12, ns)	?	
PCER Consortium (2008, Chapter 2)[5,21] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 198 total (98 intervention; 100 comparison) Mean age: 4.5 years	*Bright Beginnings* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.16, ns)	?	
			General numeracy: CMA-A, Posttest No discernible (0.14, ns)	?	
			Geometry: Shape Composition, Posttest No discernible (–0.03, ns)	?	
PCER Consortium (2008, Chapter 3)[22] RCT Meets evidence standards with reservations	Preschoolers attending Head Start centers Children: 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III– Applied Problems, Posttest No discernible (0.20, ns)	?	
			General numeracy: CMA-A Mathematics Composite, Posttest No discernible (–0.10, ns)	?	
			Geometry: Shape Composition, Posttest No discernible (0.19, ns)	?	
			Operations: WJ-III–Applied Problems, Maintenance (spring of kindergarten year) No discernible (0.09, ns)	?	
			General numeracy: CMA-A–Mathematics Composite, Maintenance (spring of kindergarten year) No discernible (0.14, ns)	?	
			Geometry: Shape Composition, Maintenance (spring of kindergarten year) No discernible (–0.01, ns)	?	

(continued)

Table D.6. Studies of interventions that used a deliberate progress-monitoring process and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested	
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Use Progress Monitoring to Tailor Instruction**	**Start with a Child's Informal Math Knowledge**
Sarama et al. (2008)[23] RCT Meets evidence standards without reservations	Head Start or state-funded prekindergarten classrooms in New York and California Children: 200 total (104 intervention; 96 comparison) Average age: 4.3 years	*Building Blocks* combined with *Pre-K Mathematics* vs. regular classroom instruction	General numeracy: REMA Positive (0.62*)	?	?

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that incorporated regular progress monitoring and emphasized using a child's existing informal math knowledge as a starting point.

X The intervention included this component.

BB Assessment = Building Blocks Assessment of Early Mathematics[343]
REMA = Research-Based Early Math Assessment[344]
CMA = Child Math Assessment[345]
CMA-A = Child Math Assessment–Abbreviated[346]
NSB = Number Sense Brief[347]
LE = Learning Express[348]
WJ-III = Woodcock-Johnson, third edition[349]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" effects are findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] **Clements and Sarama (2007b)** also reported scores for the subscales of the Number and Geometry scales; positive effects were seen for each subscale. Findings from **Clements and Sarama (2007b)** were previously reported in the WWC intervention report on *SRA Real Math Building Blocks PreK*. The panel reports the same findings as presented in the intervention report.

[7] In **Clements and Sarama (2007b)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the curricula used in the comparison classrooms, including *Creative Curriculum*, a branded comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that incorporated regular progress monitoring and encouraged using children's existing informal math knowledge as a starting point. The comparison group participated in a variety of curricula, including *Creative Curriculum*, which included progress monitoring but did not appear to encourage using children's existing informal math knowledge as a starting point.

[8] For **Clements and Sarama (2008)**, the WWC is reporting author-reported effect sizes consistent with prior reporting of findings from this study in the WWC intervention report on *SRA Real Math Building Blocks PreK*.

[9] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[10] **Clements et al. (2011)** also reported the subscale scores from the Early Mathematics Assessment. Findings for the subscale scores are consistent with the total score findings and are generally positive (9 of 13 scores). No discernible effects are seen for 4 of the 13 subscale scores (transformations/turns, comparing shapes, arithmetic, and composition of number).

[11] In **Clements et al. (2011)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the various branded curricula used in the comparison classrooms, including *DLM Early Childhood Express*, a comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that incorporated regular progress monitoring and encouraged using children's existing knowledge as a starting point. The comparison group participated in a number of branded curricula, including *DLM Early Childhood Express*, which included progress monitoring but did not appear to emphasize starting with a child's informal math knowledge to the same extent as *Building Blocks*.

[12] **Dyson, Jordan, and Glutting (2013)** reported total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all subscales at posttest and maintenance, except for the WJ-III–Applied Problems subscale, for which no discernible effects were seen at posttest or maintenance.

[13] In **Dyson, Jordan, and Glutting (2013)**, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, generally 3 a week, for a total of 24 sessions (or 12 hours). The sessions included using deliberate progress monitoring to tailor instruction. The comparison group did not receive this additional instruction; rather, they received only the regular classroom math instruction. The regular classroom math instruction, for both the intervention and comparison children, was *Math Trailblazers*, a branded math curriculum that uses deliberate progress monitoring

[14] **Fantuzzo, Gadsden, and McDermott (2011)** reported on four waves of data collection. The panel decided to use Wave 1 as pretest data, because it was collected prior to the delivery of math content. Wave 4 was used as the posttest, as it was collected at the end of the school year and delivery of the intervention. Waves 2 and 3 could be viewed as intermediary outcomes, but the panel chose to focus on posttests when determining levels of evidence.

[15] In **Fantuzzo, Gadsden, and McDermott (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *EPIC* and *DLM Early Childhood Express*, a branded comprehensive early childhood curriculum. Both curricula used progress monitoring and encouraged starting with a child's informal math knowledge.

[16] **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)** reported other outcomes for which no pretest data were provided. The WWC was unable to conduct a review that included these outcomes, as baseline equivalence could not be established.

[17] **Jordan et al. (2012)** reported posttest and maintenance effects for total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all but seven of the NSB outcomes that were reported as no discernible effects.

[18] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The sessions included instruction that used deliberate progress monitoring to tailor instruction. The comparison group did not receive this additional instruction in math; rather, they received only the regular classroom instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*. Both of these are commercially available curricula. The panel confirmed that *Math Trailblazer*s uses progress monitoring but could not confirm whether *Math Connects* includes deliberate progress monitoring.

[19] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The sessions included instruction that used deliberate progress monitoring to tailor instruction. The comparison group did not receive this additional instruction in math; rather, they received only the regular classroom instruction and additional literacy instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*. Both of these are commercially available curricula. The panel confirmed that *Math Trailblazers* uses progress monitoring but could not confirm whether *Math Connects* includes deliberate progress monitoring.

[20] In **Klein et al. (2008)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between the combined *Pre-K Mathematics* and *DLM Early Childhood Express* intervention and the curricula used in the comparison classrooms, including *Creative Curriculum*. The intervention group, which participated in a combination of *Pre-K Mathematics* and *DLM Early Childhood Express*, incorporated regular progress monitoring and emphasized using children's existing knowledge as a starting point for instruction. The comparison group participated in a number of branded curricula, including *Creative Curriculum*, a comprehensive early childhood curriculum that included progress monitoring but did not appear to emphasize starting with a child's informal knowledge in the same manner as the intervention group curricula.

[21] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel rated the study differently but reports the same findings as presented in the intervention report. The difference in study rating is due to the use of WWC Version 2.1 standards as opposed to WWC Version 1.0 standards. Findings from this study of *Bright Beginnings* were previously reported in the WWC intervention report on *Bright Beginnings*. The panel reports the same findings as reported in the intervention report. For both *Creative Curriculum* and *Bright Beginnings*, the authors report on additional outcomes that were assessed in the spring of kindergarten.

[22] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel reports the same findings as presented in the intervention report.

[23] **Sarama et al. (2008)** reported subscale scores as well; however, only the means were provided, so the WWC was unable to calculate effect sizes for the subscales.

Recommendation 4: Teach children to view and describe their world mathematically.

Level of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 14 randomized controlled trials[350] and 2 quasi-experimental studies[351] that met WWC standards and examined the effects of interventions that help children view and describe their world mathematically (see Table D.7). The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms. The studies reported both positive and no discernible effects in the outcome domains of general numeracy[352] and geometry.[353] Only positive effects were found in the outcome domain of basic number concepts.[354] One study found both positive and negative effects in the operations outcome domain.[355]

The panel believes that the most effective implementation of Recommendation 4 includes deliberate introduction of math vocabulary, creation of opportunities for children to talk about math concepts and math problem solving with one another as well as adults, and experiences that support children in linking their informal knowledge of math to formal representations of math. The 16 studies examined interventions that included key components of other recommendations as well, making it difficult to attribute the demonstrated effects to the activities related to teaching children to view and describe their world mathematically.[356] Additionally, the panel cautions that the difference in the experiences of the intervention and comparison groups with respect to math-related vocabulary and conversation may not be large enough to consider the studies a direct test of Recommendation 4. Based on their expertise and the effects of interventions that include efforts to teach children to view and describe their world mathematically, the panel believes the studies generally support this recommendation, despite the limitations to the body of evidence.

The interventions examined in each of the 16 studies included guidance for teachers and/or activities that, if implemented, would support children in learning how to view and describe their world mathematically. However, the intervention groups also participated in instructional activities that were good examples of the practices addressed in other recommendations in the practice guide. For example, to teach math vocabulary and encourage math conversation, teachers need to teach the early math content areas that are the focus of Recommendations 1 and 2. Twelve of the 16 studies included key components of Recommendation 3.[357] Fourteen of the 16 studies also included key components of Recommendation 5.[358] Finding positive effects in interventions with co-occurrence of key components of multiple recommendations supports the panel's belief that children's math achievement will improve when they are exposed to instruction that includes most, or all, of the core elements for all five recommendations.

Further, it is difficult to directly test the implementation of specific vocabulary or communication activities, because teaching academic vocabulary and encouraging communication are core activities for preschool, prekindergarten, and kindergarten classrooms, regardless of the subject matter.[359] The panel identified four studies in which the intervention group appears to have received additional instruction that encouraged the use of math vocabulary or math conversations.[360] Positive effects were reported in the domains of general numeracy and operations in two of the four studies.[361] Both positive and no discernible effects in general numeracy and operations were found in a third study.[362] The final study found both positive and negative effects in operations, depending upon the particular type of feedback the intervention and comparison groups received.[363] The amount of math vocabulary and math conversation, as well as the degree to which instruction deliberately linked informal math knowledge to formal math representations, was not clear for the comparison group in 9 of the 16 studies.[364] This group of studies

reported mixed effects in the outcome domains of general numeracy[365] and geometry[366] and only positive effects in the outcome domain of basic number concepts.[367] Three studies reported no discernible effects in the operations outcome domain.[368] The panel determined that the comparison group had participated in an intervention with core elements of Recommendation 4 in three studies[369] that found positive effects in general numeracy,[370] basic number concepts,[371] and geometry.[372]

Despite the limitations of the body of evidence for this recommendation, the panel believes—based on its own expertise and the presence of these practices in multiple studies with positive effects on math outcomes—that teaching math vocabulary and providing children with opportunities to talk about math are important for the development of children's early math skills.

Seven of the interventions provided specific math vocabulary words, and they frequently provided suggestions for stories, songs, or questions that supported children in learning to view and describe their world mathematically.[373] For example, a *Pre-K Mathematics* activity with a focus on constructing shapes identified key mathematical language including "shape," "triangle," "angle," and "five sides." *EPIC* recommended using stories, to help children learn concepts of "more" and "less" by counting animals. In *Building Blocks*, teachers were encouraged to emphasize discussion of children's solution strategies, asking questions such as "How did you know?" and "Why?" to help identify the math strategies that children were using.

Math conversation, whether with a peer or an adult, was found to be related to higher math achievement in two studies.[374] One study encouraged math conversation through the use of peer-assisted learning strategies (PALS).[375] Children were placed in mixed-ability pairs, with the stronger-performing child of the pair first serving as the coach, and then the children switching roles midway through the activity. Teachers taught children to use a correction procedure and to assist each other if one member of the pair expressed confusion. This strategy encouraged children to talk about math while they were working on it. The authors examined two general numeracy outcomes with mixed effects. Children who participated in PALS scored higher on the Stanford Early School Achievement Test (SESAT) than children who were taught the same math material (a district curriculum) without using the PALS method.[376] However, no discernible effects were found on the Primary 1 level of the Stanford Achievement Test (SAT-P).[377] In another study, children solved problems with an adult who provided feedback to each child on his or her solution. Children either explained their own response or listened to the adult's reasoning.[378] Children who received feedback and an explanation of the adult's solution scored higher, on average, than children who received only feedback.

Providing math vocabulary and encouraging communication is a part of teaching children to view and describe their world mathematically. Another important element of helping children view and describe their world mathematically is helping children link their informal knowledge with formal representations of math.[379] *Building Blocks*, a curriculum that uses learning trajectories to guide instruction, includes activities that support building the linkage between a child's informal math knowledge and formal math representations and knowledge.[380] For example, children match numeral cards with cards on which dots display the quantity that the numeral represents. This task can help children understand the relationship between the dots (an informal representation) and the numeral (a formal representation). Children participating in the *Building Blocks* intervention score, on average, higher than comparison children in general numeracy, basic number concepts, and geometry outcomes. *EPIC* is similar to *Building Blocks* in that the introduction of concepts is based on a scope and sequence. For example, children learn to use the words "more" and "less" to compare sets of objects. Children are also taught to combine sets.

Together, these activities prepare children to be introduced to the formal concepts of addition and subtraction, including the use of formal symbols to represent the operations. One study found that children participating in *EPIC* scored better, on average, than other Head Start children participating in regular classroom instruction (using *DLM Early Childhood Express*), on an assessment of general numeracy.[381]

Table D.7. Studies of interventions that incorporated math communication, math vocabulary, and linking informal knowledge to formal knowledge and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Encourage Math Communication**	**Use Math Vocabulary**	**Link Informal Knowledge to Formal Representations**
Arnold et al. (2002)[5] RCT Meets evidence standards without reservations	Pairs of half-day or full-day Head Start classes Children: 103 total (49 intervention; 54 comparison) Age range: 3.1 to 5.3 years Average age: 4.4 years (SD 7.32 months)	*Math Is Everywhere* vs. regular classroom instruction	General numeracy: TEMA-2 Positive (0.40, ns)			?
Barnett et al. (2008) RCT Meets evidence standards with reservations	Children attending a full-day preschool program Children: 202 total (85 intervention; 117 comparison) Age range: 3 to 4 years; slightly more 4-year-olds (54%)	*Tools of the Mind* vs. regular classroom instruction (district-created, balanced literacy)	Operations: WJ-Revised–Applied Math Problems Subtest No discernible (0.17, ns)	X[6]	X[6]	
Clements and Sarama (2007b)[5,7] RCT Meets evidence standards with reservations	Preschool classrooms in state-funded or Head Start programs Children: 68 total (30 intervention; 38 comparison) Age range: 2.9 to 4.8 years Mean age: 4.2 years (SD 6.2 months)	*Building Blocks* vs. regular classroom instruction (*Creative Curriculum*, locally developed)	Basic number concepts: BB Assessment–Number Scale Positive (0.75*)	X[8]	X[8]	X[8]
			Geometry: BB Assessment–Geometry Scale Positive (1.40*)	X[8]	X[8]	X[8]
Clements and Sarama (2008)[5,9] RCT Meets evidence standards without reservations	24 teachers in Head Start or state-funded preschool programs were randomly assigned to one of three conditions. 20 teachers in programs serving low- and middle-income students were randomly assigned to one of two conditions. Children: 201 total (101 intervention; 100 comparison) Children had to be within kindergarten entry range for the following year.	*Building Blocks* vs. regular classroom instruction (locally developed)	General numeracy: REMA Positive (1.07*)	?	?	?

(continued)

Table D.7. Studies of interventions that incorporated math communication, math vocabulary, and linking informal knowledge to formal knowledge and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Encourage Math Communication**	**Use Math Vocabulary**	**Link Informal Knowledge to Formal Representations**
Clements et al. (2011)[5,10,11] RCT Meets evidence standards without reservations	Prekindergarten classrooms in two urban public school districts Children: 1,305 total (927 intervention; 378 comparison)	*Building Blocks* vs. regular classroom instruction (*Where Bright Futures Begin*; *Opening the World of Learning*; *Investigations in Number, Data, and Space*; *DLM Early Childhood Express*)	General numeracy: REMA–Total Positive (0.48*)	X[12]	X[12]	X[12]
			Basic number concepts: REMA–Numbers Total Positive (0.39*)	X[12]	X[12]	X[12]
			Geometry: REMA–Geometry Total Positive (0.64*)	X[12]	X[12]	X[12]
Dyson, Jordan, and Glutting (2013)[13] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 121 total (56 intervention; 65 comparison) Mean age: 5.5 years (SD 4.0 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers*)	General numeracy: NSB–Total Score, Posttest Positive (0.64*)	X[14]	X[14]	X[14]
			Operations: WJ-III–Total Score, Posttest Positive (0.29, ns)	X[14]	X[14]	X[14]
			General numeracy: NSB–Total Score, Maintenance (6 weeks) Positive (0.65*)	X[14]	X[14]	X[14]
			Operations: WJ-III–Total Score, Maintenance (6 weeks) No discernible (0.18, ns)	X[14]	X[14]	X[14]
Fantuzzo, Gadsden, and McDermott (2011)[15] RCT Meets evidence standards without reservations	80 Head Start classrooms in Philadelphia, Pennsylvania Children: 778 total (397 intervention; 381 comparison) Age range: 2.9 to 5.8 years Mean age: 4.2 years (SD 6.8 months)	*Evidence-based Program for Integrated Curricula (EPIC)* vs. regular classroom instruction (*DLM Early Childhood Express*)	General numeracy: LE–Mathematics, Wave 4 Positive (0.18*)	X[16]	X[16]	X[16]
Fuchs, L. S., Fuchs, D., and Karns (2001)[5,17] RCT Meets evidence standards without reservations	Kindergarten teachers in Title I and non–Title I schools in a southeastern metropolitan public school system Children: 162 total (79 intervention; 83 comparison)	Peer-assisted learning strategies (PALS) vs. regular classroom instruction (same curriculum as PALS, a district curriculum including *Math Advantage Grade K Basal*)	General numeracy: SESAT Positive (0.28, ns)	X[18]		
			General numeracy: SAT-P No discernible (0.12, ns)	X[18]		

(continued)

Table D.7. Studies of interventions that incorporated math communication, math vocabulary, and linking informal knowledge to formal knowledge and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Encourage Math Communication**	**Use Math Vocabulary**	**Link Informal Knowledge to Formal Representations**
Griffin, Case, and Capodilupo (1995) and related publication Griffin, Case, and Siegler (1994)[19] QED Meets evidence standards with reservations	Kindergarten students in public schools in inner-city areas in Massachusetts Children: 47 total (23 intervention; 24 comparison)	*Rightstart* vs. regular classroom instruction	Basic number concepts: NKT Positive (1.79*)	?		
Jordan et al. (2012)[20] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 86 total (42 intervention; 44 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers* or *Math Connects*)	General numeracy: NSB–Total, Posttest Positive (1.10*)	X[21]	X[21]	X[21]
			Operations: WJ-III–Total, Posttest Positive (0.91*)	X[21]	X[21]	X[21]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.77*)	X[21]	X[21]	X[21]
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.56*)	X[21]	X[21]	X[21]
Jordan et al. (2012)[20] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 84 total (42 intervention; 42 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. treated comparison (supplemental language intervention with *Math Trailblazers* or *Math Connects*)	General numeracy: NSB, Total, Posttest Positive (0.91*)	X[22]	X[22]	X[22]
			Operations: WJ-III–Total, Posttest Positive (0.84*)	X[22]	X[22]	X[22]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.62*)	X[22]	X[22]	X[22]
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.75*)	X[22]	X[22]	X[22]
Klein et al. (2008)[5] RCT Meets evidence standards without reservations	40 prekindergarten classrooms in Head Start or state-funded programs in New York and California Children: 278 total (138 intervention; 140 comparison) Age range: 3.8 to 4.9 years Mean age: 4.4 years	*Pre-K Mathematics* combined with *DLM Early Childhood Express* vs. regular classroom instruction (*Creative Curriculum*, *High Scope*, Montessori, locally developed)	General numeracy: CMA Positive (0.57*)	X[23]	X[23]	X[23]

(continued)

Table D.7. Studies of interventions that incorporated math communication, math vocabulary, and linking informal knowledge to formal knowledge and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Encourage Math Communication**	**Use Math Vocabulary**	**Link Informal Knowledge to Formal Representations**
PCER Consortium (2008, Chapter 2)[5,24] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 193 total (93 intervention; 100 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.17, ns)	?	?	
			General numeracy: CMA-A, Posttest No discernible (0.10, ns)	?	?	
			Geometry: Shape Composition, Posttest No discernible (–0.12, ns)	?	?	
PCER Consortium (2008, Chapter 2)[5,24] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 198 total (98 intervention; 100 comparison) Mean age: 4.5 years	*Bright Beginnings* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.16, ns)	?	?	
			General numeracy: CMA-A, Posttest No discernible (0.14, ns)	?	?	
			Geometry: Shape Composition, Posttest No discernible (–0.03, ns)	?	?	
PCER Consortium (2008, Chapter 3)[25] RCT Meets evidence standards with reservations	Preschoolers attending Head Start centers Children: 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.20, ns)	?	?	
			General numeracy: CMA-A–Mathematics Composite, Posttest No discernible (–0.10, ns)	?	?	
			Geometry: Shape Composition, Posttest No discernible (0.19, ns)	?	?	
			Operations: WJ-III–Applied Problems, Maintenance (spring of kindergarten year) No discernible (0.09, ns)	?	?	
			General numeracy: CMA-A–Mathematics Composite, Maintenance (spring of kindergarten year) No discernible (0.14, ns)	?	?	
			Geometry: Shape Composition, Maintenance (spring of kindergarten year) No discernible (–0.01, ns)	?	?	

(continued)

Table D.7. Studies of interventions that incorporated math communication, math vocabulary, and linking informal knowledge to formal knowledge and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Encourage Math Communication**	**Use Math Vocabulary**	**Link Informal Knowledge to Formal Representations**
Sarama et al. (2008)[26] RCT Meets evidence standards without reservations	Head Start or state-funded prekindergarten classrooms in New York and California Children: 200 total (104 intervention; 96 comparison) Average age: 4.3 years	*Building Blocks* combined with *Pre-K Mathematics* vs. regular classroom instruction	General numeracy: REMA Positive (0.62*)	?	?	?
Siegler (1995)[10] RCT Meets evidence standards without reservations	University-based preschool, university-based day care center, or day care center in a middle-class community Children: 30 total (15 intervention; 15 comparison) Age range: 4.5 to 6.1 years Mean age: 5.3 years	Feedback with explanation of own reasoning vs. treated comparison (feedback only)	Operations: Percent Correct Judgments Negative (–0.60, ns)	X[27]	X[27]	
Siegler (1995)[10] RCT Meets evidence standards without reservations	University-based preschool, university-based day care center, or day care center in a middle-class community Children: 30 total (15 intervention; 15 comparison) Age range: 4.5 to 6.1 years Mean age: 5.3 years	Feedback with explanation of rater's reasoning vs. treated comparison (feedback only)	Operations: Percent Correct Judgments Positive (0.30, ns)	X[28]	X[28]	
Siegler (1995)[10] RCT Meets evidence standards without reservations	University-based preschool, university-based day care center, or day care center in a middle-class community Children: 30 total (15 intervention; 15 comparison) Age range: 4.5 to 6.1 years Mean age: 5.3 years	Feedback with explanation of own reasoning vs. treated comparison (feedback with explanation of rater's reasoning)	Operations: Percent Correct Judgments Negative (–0.88*)	X[29]	X[29]	
Sophian (2004)[5,10] QED Meets evidence standards with reservations	Head Start sites Children: 94 total (46 intervention; 48 comparison) Age range: 2 years, 6 months to 4 years, 7 months	Researcher-developed, measurement-focused curriculum vs. treated comparison (literacy instruction)	General numeracy: DSC–Mathematics Subscale Positive (0.33, ns)	X[30]	X[30]	X[30]

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that taught math vocabulary, encouraged communication about math, and supported children in linking informal and formal math knowledge.

X The intervention included this component.

BB Assessment = Building Blocks Assessment of Early Mathematics[382]
CMA = Child Math Assessment[383]
CMA-A = Child Math Assessment–Abbreviated[384]
SESAT = Stanford 7 Plus[385]
SAT-P = Stanford Achievement Test–Primary 1[386]
NSB = Number Sense Brief[387]

REMA = Research-Based Early Math Assessment[388]
WJ-Revised = Woodcock-Johnson, revised edition[389]
WJ-III = Woodcock-Johnson, third edition[390]
DSC = Developing Skills Checklist[391]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) the type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" effects are findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] In **Barnett et al. (2008)**, the difference between the intervention and comparison groups with respect to math instruction is not known. The intervention group participated in *Tools of the Mind*, a comprehensive early childhood curriculum with a math component that supported incorporating math into other parts of the school day. The comparison group participated in a district-created balanced literacy curriculum. From the information provided, it was not clear how the intervention and comparison groups differed with respect to teaching children to use math vocabulary or encouraging them to communicate about math.

[7] **Clements and Sarama (2007b)** also reported scores for the subscales of the Number and Geometry scales; positive effects were seen for each subscale. Findings from **Clements and Sarama (2007b)** were previously reported in the WWC intervention report on *SRA Real Math Building Blocks PreK*. The panel reports the same findings as presented in the intervention report.

[8] In **Clements and Sarama (2007b)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the curricula used in the comparison classrooms, including *Creative Curriculum*, a branded comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that taught children to view and describe their world mathematically. *Building Blocks* included teaching math vocabulary, encouraging communication about math, and supporting children in linking informal and formal math knowledge. The comparison group participated in a variety of curricula, including *Creative Curriculum*, which taught math vocabulary and encouraged communication about math but did not appear to support linking informal and formal knowledge.

[9] For **Clements and Sarama (2008)**, the WWC is reporting author-reported effect sizes consistent with prior reporting of findings from this study in the WWC intervention report on *SRA Real Math Building Blocks PreK*.

[10] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[11] **Clements et al. (2011)** also reported the subscale scores from the REMA. Findings for the subscale scores were consistent with the total score findings and were generally positive (9 of 13 scores). No discernible effects were seen for 4 of the 13 subscale scores (two in the geometry domain: transformations/turns and comparing shapes; one in the operations domain: arithmetic; and one in the basic number concepts domain: composition of number).

[12] In **Clements et al. (2011)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the various branded curricula used in the comparison classrooms, including *DLM Early Childhood Express*, a comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that taught children to view and describe their world mathematically. *Building Blocks* included teaching math vocabulary, encouraging communication about math, and supporting children in linking informal and formal math knowledge. The comparison group participated in a number of branded curricula, including *DLM Early Childhood Express*, which taught math vocabulary, encouraged communication about math, and supported children in linking informal and formal knowledge.

[13] **Dyson, Jordan, and Glutting (2013)** reported total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all subscales at posttest and maintenance, except for the WJ-III–Applied Problems subscale, for which no discernible effects were seen at posttest or maintenance.

[14] In **Dyson, Jordan, and Glutting (2013)**, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, generally 3 a week, for a total of 24 sessions (or 12 hours). The intervention group participated in additional number sense instruction that included teaching math vocabulary, encouraging communication about math, and supporting children in linking informal and formal knowledge. The comparison group did not receive this additional instruction; rather, they received only the regular classroom math instruction. The regular classroom math instruction, for both the intervention and comparison children, was *Math Trailblazers*, a branded math curriculum used to teach number and operations but not guided by a developmental progression.

[15] **Fantuzzo, Gadsden, and McDermott (2011)** reported on four waves of data collection. The panel decided to use Wave 1 as pretest data, because it was collected prior to the delivery of math content. Wave 4 was used as the posttest, as it was collected at the end

of the school year and delivery of the intervention. Waves 2 and 3 could be viewed as intermediary outcomes, but the panel chose to focus on posttests when determining levels of evidence.

[16] In **Fantuzzo, Gadsden, and McDermott (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *EPIC* and *DLM Early Childhood Express*, a branded comprehensive early childhood curriculum. Both curricula taught math vocabulary, encouraged communication about math, and supported linking informal and formal math knowledge.

[17] **Fuchs, L. S., Fuchs, D., and Karns (2001)** did not provide a pretest for the SAT-P. The panel decided to use the SESAT pretest for the post-hoc difference-in-difference adjustment.

[18] In **Fuchs, L. S., Fuchs, D., and Karns (2001)**, the difference between the intervention and comparison groups was the use of peer-assisted learning strategies to practice math problem solving. Both the intervention and comparison groups participated in similar math instruction using the district curriculum, which included the *Math Advantage Grade K Basal*. The intervention group also took turns working in pairs, with both children serving as "coach" while solving math problems together; this provided an opportunity for children to practice communicating about math with peers. The comparison group did not participate in any peer-assisted learning strategies to practice math skills.

[19] **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)** reported other outcomes for which no pretest data were provided. The WWC was unable to conduct a review that included these outcomes, as baseline equivalence could not be established.

[20] **Jordan et al. (2012)** reported posttest and maintenance effects for total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all but seven of the NSB outcomes that were reported as no discernible effects.

[21] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The intervention group participated in additional number sense instruction that included teaching math vocabulary, encouraging communication about math, and supporting children in linking informal and formal knowledge. The comparison group did not receive this additional instruction in math; rather, they only had the regular classroom instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*, both of which are commercially available curricula.

[22] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The intervention group participated in additional number sense instruction that included teaching math vocabulary, encouraging communication about math, and supporting children in linking informal and formal knowledge. The comparison group did not receive this additional instruction in math; rather, they received only the regular classroom instruction and additional literacy instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*, both of which are commercially available curricula.

[23] In **Klein et al. (2011)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between the combined *Pre-K Mathematics* and *DLM Early Childhood Express* intervention and the curricula used in the comparison classrooms, including *Creative Curriculum*, a branded comprehensive early childhood curriculum. The intervention group participated in a combination of *Pre-K Mathematics* and *DLM Early Childhood Express*, which taught math vocabulary, encouraged communication about math, and supported children in linking informal and formal knowledge. The comparison group participated in a number of branded curricula, including *Creative Curriculum*, a comprehensive early childhood curriculum that included regular math lessons.

[24] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel rated the study differently but reports the same findings as presented in the intervention report. The difference in study rating is due to the use of WWC Version 2.1 standards as opposed to WWC Version 1.0 standards. Findings from this study of *Bright Beginnings* were previously reported in the WWC intervention report on *Bright Beginnings*. The panel reports the same findings as reported in the intervention report. For both *Creative Curriculum* and *Bright Beginnings*, the authors reported on additional outcomes that were assessed in the spring of kindergarten.

[25] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel reports the same findings as presented in the intervention report.

[26] **Sarama et al. (2008)** reported subscale scores as well; however, only the means were provided, so the WWC was unable to calculate effect sizes for the subscales.

[27] There are three comparisons in **Siegler (1995)**. In this comparison, the difference between the intervention and comparison groups was the explanation of the solution provided in conjunction with feedback. Both groups received feedback on their response. The children in the intervention group provided an explanation of their own reasoning. Children in the comparison group did not provide or receive any explanation.

[28] There are three comparisons in **Siegler (1995)**. In this comparison, the difference between the intervention and comparison groups was the explanation of the solution provided in conjunction with feedback. Both groups received feedback on their response. Children in the intervention group listened to the rater's explanation of their response. Children in the comparison group did not provide or receive any explanation.

[29] There are three comparisons in **Siegler (1995)**. In this comparison, the difference between the intervention and comparison groups was the explanation of the solution provided in conjunction with feedback. Both groups received feedback on their response. Children in the intervention group provided an explanation of their own reasoning. Children in the comparison group listened to the rater's explanation of their reasoning.

[30] In **Sophian (2004)**, the difference between the intervention and comparison groups was whether children received math instruction using a researcher-developed, measurement-focused curriculum. The intervention group participated in a researcher-developed, measurement-focused curriculum that emphasized the concept of unit, provided math vocabulary, encouraged communication about math, and supported children in linking informal and formal knowledge. The comparison group participated in a literacy curriculum. There is no description of the math instruction children in the comparison group may have received as part of their regular classroom instruction.

Recommendation 5: Dedicate time each day to teaching math, and integrate math throughout the school day.

Level of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 18 randomized controlled trials[392] and 2 quasi-experimental studies[393] that met WWC standards and examined the effects of interventions that included dedicated time for math instruction, integrating math in other aspects of the school day, and playing games to practice math skills (see Table D.8). Children in the studies attended preschool, prekindergarten, and kindergarten classrooms. Only positive findings were found in the domain of basic number concepts.[394] Both positive and no discernible effects were found in the outcome domains of general numeracy,[395] number recognition,[396] operations,[397] and geometry.[398]

The panel believes that teachers should dedicate time to math instruction daily as well as take advantage of opportunities to integrate math into other classroom activities, including games and instruction in other content areas. Math instruction was a regular, if not daily, activity for the intervention groups in 14 of the 20 studies.[399] Integration of math into other content areas was a focus in the interventions examined in 11 of the 20 studies.[400] The panel was able to determine that 6 of the 20 studies deliberately played games to reinforce math skills.[401]

The panel identified two areas of concern regarding the evidence associated with this recommendation. First, the interventions examined always included key elements of other recommendations (i.e., they were multi-component interventions).[402] Thus, the panel was unable to attribute the effects seen to the instruction of math both at specific points during the day and during instruction in other content areas. Second, since many preschool, prekindergarten, and kindergarten classrooms are teaching math either as a particular subject or in conjunction with other content areas,[403] the panel determined that it was highly unlikely that the comparison groups were receiving no math instruction. For this reason, the panel did not consider the studies to be direct tests of Recommendation 5. Based on their expertise and the effects of interventions that include dedicated time each day to teach math and/or efforts to integrate math instruction throughout the school day, the panel believes the studies generally support this recommendation despite the limitations of the body of evidence.

The panel determined that in 7 of the 20 studies, the intervention group received more math instruction (including playing games) than the comparison group.[404] Both positive and no discernible effects were found in the outcome domains of general numeracy, basic number concepts, number recognition, and operations.[405] In 11 of the 20 studies, the difference between the intervention and comparison groups could not be definitively identified based on the information provided in the studies.[406] The panel concluded that although the intervention group clearly received components of Recommendation 5, the comparison group may have also participated in instruction that included components of Recommendation 5. These 11 studies reported positive effects in the outcome domains of general numeracy,[407] basic number concepts,[408] and geometry.[409] No discernible effects were also reported by these studies in the outcome domains of general numeracy,[410] operations,[411] and geometry.[412] In the final two studies, the intervention and comparison groups both received components of Recommendation 5;[413] positive effects were found in general numeracy,[414] basic number concepts,[415] and geometry.[416]

All 20 studies the panel reviewed included components of Recommendation 5. However, the panel does not consider the evidence sufficient to warrant a rating of moderate due, to the presence of key components from all recommendations in the examined interventions

and the differences between the intervention and comparison groups. Every intervention examined for this recommendation included components of other recommendations—for example, targeted instruction in number and operations, geometry, patterns, measurement, and data analysis—which may have contributed to the overall effects seen. Furthermore, based on their own experiences, the members of panel recognize that it is rare to find a preschool, prekindergarten, or kindergarten classroom that is not doing some sort of math activity. However, the panel believes that children in classrooms that both provide regular "math time" and integrate math into other content areas will learn more math than children in classrooms that do not include these experiences.

The interventions investigated in these studies provided teachers with opportunities to implement instructional activities described as part of this recommendation; however, many of the interventions included instructional activities that are key components of other recommendations in the practice guide. This overlap is not surprising to the panel, as this recommendation focuses on situations in which math activities could be implemented (e.g., daily classroom routines), as well as the methods teachers should use to reinforce and extend early math concepts and skills (e.g., board games). Other recommendations in the guide, Recommendations 1 and 2 in particular, focus directly on the early math content areas that should be taught to preschool, prekindergarten, and kindergarten children; hence, these studies also support those recommendations. However, based on panel members' own expertise and the presence of these activities in multiple studies showing positive effects on math outcomes, the panel believes that it is important to the development of children's early math skills for teachers to include activities that reinforce and extend early math concepts and skills.

The interventions examined in this body of evidence are of three types: (1) interventions that specifically focused on providing activities that allowed teachers to integrate math into everyday situations and routines; (2) interventions that provided activities a teacher could implement as a part of a larger curriculum; and (3) interventions that used board games to increase children's math competence and skills.

The panel identified one study in which the intervention was developed specifically to reinforce early math concepts and skills in everyday situations.[417] The curriculum, *Math Is Everywhere*, is a collection of 85 suggested activities using a variety of approaches (e.g., books, music, games, discussions, and group projects). It provided teachers with specific activities that reinforce math concepts and could be implemented during various times of the day. In the study, the curriculum was implemented in preschool classrooms over a 6-week period. During the first 3 weeks, teachers were encouraged to implement at least one circle-time activity each day. As one activity, during circle time, the teacher would ask all children who have a cat to stand up, and then a child would count aloud the number of children standing. During the second 3 weeks, the teachers implemented two transition or meal-time activities and one small-group activity per day. Children in classrooms using *Math Is Everywhere* scored higher in the general numeracy domain than children in classrooms where the teachers continued their regular classroom instruction.

Two interventions, *Building Blocks* and the *Pre-K Mathematics* curriculum, as well as two researcher-developed interventions, focused on math and included activities that could be integrated into the classroom environment, including everyday routines and activities.[418] For example, *Building Blocks* provided math activities such as verbal counting or counting objects (e.g., children putting a specific number of toppings on a cookie), which could be integrated throughout the school day. Highlighting early math concepts while reading stories or using movement to reinforce children's development of skills in these areas are ideas that teachers may be able to include in their regular classroom routines.[419] Children

participating in interventions that supported reinforcing and extending math concepts in the classroom environment, routines, and other activities scored higher, on average, than children in the comparison group, in the domains of general numeracy,[420] basic number concepts,[421] and geometry.[422]

Several other studies investigated interventions in which children's math concepts were reinforced by playing board games, an activity specified in the panel's recommendation.[423] In these studies, children played *The Great Race*, a numerical board game, one-on-one with the experimenter over the course of three to four 15- to 20-minute sessions.[424] The studies generally found that children who played number-based board games performed better in the domain of basic number concepts than children who played color-based board games or no board games. Mixed effects were found in the domain of number recognition, with one study finding positive effects on children's performance, one study finding no discernible effects, and a third study finding both positive and no discernible effects. These studies also found positive and no discernible effects in the domain of operations. Two other studies included *The Great Race* as a part of a number sense intervention.[425] Both studies found positive effects on outcomes in the general numeracy and operations domain at posttest. That is, children who participated in the number sense curriculum—including playing *The Great Race*—scored higher, on average, than children receiving regular classroom instruction or children in the treated comparison group, who participated in a literacy intervention.

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Arnold et al. (2002)[5] RCT Meets evidence standards without reservations	Pairs of half-day or full-day Head Start classes Children: 103 total (49 intervention; 54 comparison) Age range: 3.1 to 5.3 years Average age: 4.4 years (SD 7.32 months)	*Math Is Everywhere* vs. regular classroom instruction	General numeracy: TEMA-2 Positive (0.40, ns)	?	X[6]	
Aunio, Hautamaki, and Van Luit (2005)[5,7] RCT Meets evidence standards without reservations	Pairs of matched students attending two large preschools in Helsinki, Finland, were randomly assigned. Four smaller preschools in Helsinki, Finland, were randomly assigned. Children: 45 total (22 intervention; 23 comparison) Age range: 4.7 to 6.6 years Mean age: 5.5 years (SD 6.4 months)	*Let's Think!* combined with *Maths!* vs. regular classroom instruction	Basic number concepts: ENT–Relational Scale, Posttest Positive (0.77, ns)	?		
			Basic number concepts: ENT–Counting Scale, Posttest Positive (0.87*)	?		
			Geometry: Geometrical Analogies, Posttest Positive (0.25, ns)	?		

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Aunio, Hautamaki, and Van Luit (2005)[5,7] RCT Meets evidence standards without reservations *(continued)*	Pairs of matched students attending two large preschools in Helsinki, Finland, were randomly assigned. Four smaller preschools in Helsinki, Finland, were randomly assigned. Children: 45 total (22 intervention; 23 comparison) Age range: 4.7 to 6.6 years Mean age: 5.5 years (SD 6.4 months)	*Let's Think!* combined with *Maths!* vs. regular classroom instruction	Geometry: SRT, Posttest No discernible (0.20, ns)	?		
			Basic number concepts: ENT–Relational Scale, Maintenance (6 months) Positive (0.48, ns)	?		
			Basic number concepts: ENT–Counting Scale, Maintenance (6 months) Positive (0.36, ns)	?		
			Geometry: Geometrical Analogies, Maintenance (6 months) No discernible (0.24, ns)	?		
			Geometry: SRT, Maintenance (6 months) Positive (0.36, ns)	?		
Barnett et al. (2008) RCT Meets evidence standards with reservations	Children attending a full-day preschool program Children: 202 total (85 intervention; 117 comparison) Age range: 3 to 4 years; slightly more 4-year-olds (54%)	*Tools of the Mind* vs. regular classroom instruction (district-created, balanced literacy)	Operations: WJ-Revised–Applied Math Problems Subtest No discernible (0.17, ns)		X[8]	
Clements and Sarama (2007b)[5,9] RCT Meets evidence standards with reservations	Preschool classrooms in state-funded or Head Start programs Children: 68 total (30 intervention; 38 comparison) Age range: 2.9 to 4.8 years Mean age: 4.2 years (SD 6.2 months)	*Building Blocks* vs. regular classroom instruction (*Creative Curriculum*, locally developed)	Basic number concepts: BB Assessment–Number Scale Positive (0.75*)	X[10]	X[10]	X[10]
			Geometry: BB Assessment–Geometry Scale Positive (1.40*)	X[10]	X[10]	X[10]
Clements and Sarama (2008)[5,11] RCT Meets evidence standards without reservations	24 teachers in Head Start or state-funded preschool programs were randomly assigned to one of three conditions. 20 teachers in programs serving low- and middle-income students were randomly assigned to one of two conditions. Children: 201 total (101 intervention; 100 comparison) Children had to be within kindergarten entry range for the following year.	*Building Blocks* vs. regular classroom instruction (locally developed)	General numeracy: REMA Positive (1.07*)	?	?	?

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Clements et al. (2011)[5,7,12] RCT Meets evidence standards without reservations	Prekindergarten classrooms in two urban public school districts Children: 1,305 total (927 intervention; 378 comparison)	*Building Blocks* vs. regular classroom instruction (*Where Bright Futures Begin*; *Opening the World of Learning*; *Investigations in Number, Data, and Space*; *DLM Early Childhood Express*)	General numeracy: REMA–Total Positive (0.48*)	X[13]	X[13]	X[13]
			Basic number concepts: REMA–Numbers Total Positive (0.39*)	X[13]	X[13]	X[13]
			Geometry: REMA–Geometry Total Positive (0.64*)	X[13]	X[13]	X[13]
Dyson, Jordan, and Glutting (2013)[14] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 121 total (56 intervention; 65 comparison) Mean age: 5.5 years (SD 4.0 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers*)	General numeracy: NSB–Total Score, Posttest Positive (0.64*)	X[15]		X[15]
			Operations: WJ-III–Total Score, Posttest Positive (0.29, ns)	X[15]		X[15]
			General numeracy: NSB–Total Score, Maintenance (6 weeks) Positive (0.65*)	X[15]		X[15]
			Operations: WJ-III–Total Score, Maintenance (6 weeks) No discernible (0.18, ns)	X[15]		X[15]
Fantuzzo, Gadsden, and McDermott (2011)[16] RCT Meets evidence standards without reservations	80 Head Start classrooms in Philadelphia, Pennsylvania Children: 778 total (397 intervention; 381 comparison) Age range: 2.9 to 5.8 years Mean age: 4.2 years (SD 6.8 months)	*Evidence-based Program for Integrated Curricula (EPIC)* vs. regular classroom instruction (*DLM Early Childhood Express*)	General numeracy: LE–Mathematics, Wave 4 Positive (0.18*)	X[17]	X[17]	
Griffin, Case, and Capodilupo (1995) and related publication Griffin, Case, and Siegler (1994)[18] QED Meets evidence standards with reservations	Kindergarten students in public schools in inner-city areas in Massachusetts Children: 47 total (23 intervention; 24 comparison)	*Rightstart* vs. regular classroom instruction	Basic number concepts: NKT Positive (1.79*)	?	?	?

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Jordan et al. (2012)[19] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 86 total (42 intervention; 44 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. regular classroom instruction (*Math Trailblazers* or *Math Connects*)	General numeracy: NSB–Total, Posttest Positive (1.10*)	X[20]		X[20]
			Operations: WJ-III–Total, Posttest Positive (0.91*)	X[20]		X[20]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.77*)	X[20]		X[20]
			Operations: WJ-III, Total, Maintenance (8 weeks) Positive (0.56*)	X[20]		X[20]
Jordan et al. (2012)[19] RCT Meets evidence standards without reservations	Kindergarten students attending full-day kindergarten in one of five schools in one district in the Mid-Atlantic region of the United States Children: 84 total (42 intervention; 42 comparison) Mean age: 5.5 years (SD 4.38 months)	Supplemental researcher-developed number sense curriculum vs. treated comparison (supplemental language intervention with *Math Trailblazers* or *Math Connects*)	General numeracy: NSB, Total, Posttest Positive (0.91*)	X[21]		X[21]
			Operations: WJ-III–Total, Posttest Positive (0.84*)	X[21]		X[21]
			General numeracy: NSB–Total, Maintenance (8 weeks) Positive (0.62*)	X[21]		X[21]
			Operations: WJ-III–Total, Maintenance (8 weeks) Positive (0.75*)	X[21]		X[21]
Klein et al. (2008)[5] RCT Meets evidence standards without reservations	40 prekindergarten classrooms in Head Start or state-funded programs in New York and California Children: 278 total (138 intervention; 140 comparison) Age range: 3.8 to 4.9 years Mean age: 4.4 years	*Pre-K Mathematics* combined with *DLM Early Childhood Express* vs. regular classroom instruction (*Creative Curriculum*, *High Scope*, Montessori, locally developed)	General numeracy: CMA Positive (0.57*)	X[22]	X[22]	X[22]
Monahan (2007)[23] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 83 total (41 intervention; 42 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math with story vs. treated comparison (math)	General numeracy: ENCO Assessment No discernible (0.03, ns)	X[24]	X[24]	

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Monahan (2007)[23] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 76 total (37 intervention; 39 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math with movement vs. treated comparison (math)	General numeracy: ENCO Assessment Positive (0.32, ns)	X[25]	X[25]	
Monahan (2007)[23] RCT Meets evidence standards without reservations	Children attending Head Start centers in Philadelphia, Pennsylvania Children: 76 total (37 intervention; 39 comparison) Age range: 4 to 6 years Mean age: 5 years, 1 month	Math with movement vs. treated comparison (math with story)	General numeracy: ENCO Assessment Positive (0.31, ns)	X[26]	X[26]	
PCER Consortium (2008, Chapter 2)[5,27] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 193 total (93 intervention; 100 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.17, ns)	?		
			General numeracy: CMA-A, Posttest No discernible (0.10, ns)	?		
			Geometry: Shape Composition, Posttest No discernible (–0.12, ns)	?		
PCER Consortium (2008, Chapter 2)[5,27] RCT Meets evidence standards with reservations	Prekindergarten teachers working in public programs the year before the study began Children: 198 total (98 intervention; 100 comparison) Mean age: 4.5 years	*Bright Beginnings* vs. classroom instruction (teacher-developed nonspecific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.16, ns)	?		
			General numeracy: CMA-A, Posttest No discernible (0.14, ns)	?		
			Geometry: Shape Composition, Posttest No discernible (–0.03, ns)	?		

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
PCER Consortium (2008, Chapter 3)[28] RCT Meets evidence standards with reservations	Preschoolers attending Head Start centers Children; 170 total (90 intervention; 80 comparison) Mean age: 4.5 years	*Creative Curriculum* vs. regular classroom instruction (teacher-developed non-specific curricula)	Operations: WJ-III–Applied Problems, Posttest No discernible (0.20, ns)	?		
			General numeracy: CMA-A, Posttest No discernible (–0.10, ns)	?		
			Geometry: Shape Composition, Posttest No discernible (0.19, ns)	?		
			Operations: WJ-III–Applied Problems, Maintenance (spring of kindergarten year) No discernible (0.09, ns)	?		
			General numeracy: CMA-A, Maintenance (spring of kindergarten year) No discernible (0.14, ns)	?		
			Geometry: Shape Composition, Maintenance (spring of kindergarten) No discernible (–0.01, ns)	?		
Ramani and Siegler (2008)[29] RCT Meets evidence standards without reservations	Preschoolers attending Head Start programs Children: 124 total (68 intervention; 56 comparison) Age range: 4 years, 1 month to 5 years, 5 months Mean age: 4 years, 9 months (SD 0.44)	Number-based board games vs. treated comparison (color-based board games)	Basic number concepts: Counting, Posttest Positive (0.74*)			X[30]
			Basic number concepts: Numerical Magnitude Comparison, Posttest Positive (0.99*)			X[30]
			Number recognition: Number Identification, Posttest Positive (0.69*)			X[30]
			Basic number concepts: Counting, Maintenance (9 weeks) Positive (0.66*)			X[30]
			Basic number concepts: Numerical Magnitude Comparison, Maintenance (9 weeks) Positive (0.77*)			X[30]
			Number recognition: Number Identification, Maintenance (9 weeks) Positive (0.80*)			X[30]

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Ramani and Siegler (2011, Experiment 1)[7] RCT Meets evidence standards without reservations	Preschoolers attending one of six preschool programs (three were affiliated with universities) Children: 88 total (30 in linear, number-based board games; 29 in circular board games; 29 in numerical activities control) Age range: 3 years, 5 months to 4 years, 8 months Mean age: 4 years (SD 0.49)	Linear, number-based board games vs. treated comparison (number string counting, numeral identification, and object counting)	Number recognition: Numeral Identification No discernible (0.24, ns)			X[32]
			Operations: Arithmetic–Percent Correct Answers No discernible (ns)[24]			X[32]
			Operations: Arithmetic–Absolute Error Positive (0.31, ns)[31]			X[32]
Ramani and Siegler (2011, Experiment 1)[7] RCT Meets evidence standards without reservations	Preschoolers attending one of six preschool programs (three were affiliated with universities) Children: 88 total (30 in linear, number-based board games; 29 in circular board games; 29 in numerical activities control) Age range: 3 years, 5 months to 4 years, 8 months Mean age: 4 years (SD 0.49)	Circular, number-based board game vs. treated comparison (number string counting, numeral identification, and object counting)	Number recognition: Numeral Identification No discernible (0.24, ns)			X[33]
			Operations: Arithmetic–Percent Correct Answers No discernible (ns)[24]			X[33]
			Operations: Arithmetic–Absolute Error Positive (0.41, ns)			X[33]
Sarama et al. (2008)[34] RCT Meets evidence standards without reservations	Head Start or state-funded prekindergarten classrooms in New York and California Children: 200 total (104 intervention; 96 comparison) Average age: 4.3 years	*Building Blocks* combined with *Pre-K Mathematics* vs. regular classroom instruction	General numeracy: REMA Positive (0.62*)	?	?	?
Siegler and Ramani (2008, Experiment 2)[29] RCT Meets evidence standards without reservations	Preschool-aged children attending Head Start or one of three childcare centers Children: 36 total (18 intervention; 18 comparison) Age range: 4 to 5.1 years Mean age: 4.6 years (SD 0.30) for linear number-based board games; 4.7 years (SD 0.42) for color-based board games	Linear number-based board games vs. treated comparison (color-based board games)	Basic number concepts: Number Line Estimation–Percent Absolute Error Positive (0.86*)[31]			X[30]
			Basic number concepts: Percent of Correctly Ordered Numbers Positive (1.17*)			X[30]

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Siegler and Ramani (2009)[7,29] RCT Meets evidence standards without reservations	Preschoolers attending Head Start programs or one of two childcare centers Children: 59 total (30 intervention; 29 comparison) Age range: 4 years to 5 years, 5 months Mean age: 4 years, 8 months (SD 0.47)	Linear number-based board games vs. treated comparison (number string counting, numeral identification, and object counting)	Basic number concepts: Number Line Estimation–Percent Absolute Error Positive (0.63*)[31]			X[36]
			Basic number concepts: Numerical Magnitude Comparison No discernible (ns)[34]			X[36]
			Basic number concepts: Counting–Percentage Correctly Counting to 10 No discernible (ns)[34]			X[36]
			Number recognition: Number Identification No discernible (ns)[34]			X[36]
			Operations: Arithmetic–Percentage Answered Correctly No discernible (ns)[34]			X[36]
			Operations: Arithmetic–Percent Absolute Error No discernible (ns)[34]			X[36]
Siegler and Ramani (2009)[7,29] RCT Meets evidence standards without reservations	Preschoolers attending Head Start programs or one of two childcare centers Children: 58 total (29 intervention; 29 comparison) Age range: 4 years to 5 years, 5 months Mean age: 4 years, 8 months (SD 0.47)	Circular number-based board games vs. treated comparison (number string counting, numeral identification, and object counting)	Basic number concepts: Number Line Estimation–Percent Absolute Error No discernible (ns)[31,34]			X[37]
			Basic number concepts: Numerical Magnitude Comparison No discernible (ns)[34]			X[37]
			Basic number concepts: Counting–Percentage Correctly Counting to 10 No discernible (ns)[34]			X[37]
			Number recognition: Number Identification No discernible (ns)[34]			X[37]
			Operations: Arithmetic–Percentage Answered Correctly No discernible (ns)[34]			X[37]
			Operations: Arithmetic–Percent Absolute Error No discernible (ns)[34]			X[37]

(continued)

Table D.8. Studies of interventions that included regular math time, incorporated math into other aspects of the school day, and used games to reinforce math skills and contributed to the level of evidence rating *(continued)*

Study Characteristics				Recommendation Components Tested		
Citation, Design, and WWC Rating[1]	**Population Characteristics[2]**	**Comparison[3]**	**Findings (Domain: Assessment (Effect Size, Significance))[4]**	**Include Regular Math Lessons**	**Incorporate Math into Other Parts of the Day**	**Use Games to Reinforce Math Skills**
Sophian (2004)[5,8] QED Meets evidence standards with reservations	Preschoolers attending Head Start programs Children: 94 total (46 intervention; 48 comparison) Age range: 2 years, 6 months to 4 years, 7 months	Researcher-developed, measurement-focused curriculum vs. treated comparison (literacy instruction)	General numeracy: DSC–Mathematics Subscale Positive (0.33, ns)		X[38]	

? There was not sufficient description of the type and nature of the instruction the comparison group received. Children in the comparison group may have participated in instruction that included regular math lessons, incorporated math into other parts of the day, or used games to reinforce math skills.

X The intervention included this component.

BB Assessment = Building Blocks Assessment of Early Mathematics[426]
REMA = Research-Based Early Math Assessment[427]
NKT = Number Knowledge Test[428]
DSC = Developing Skills Checklist[429]
LE = Learning Express[430]
WJ-Revised = Woodcock-Johnson, revised edition[431]
WJ-III = Woodcock-Johnson, third edition[432]
CMA = Child Math Assessment[433]
TEMA-2 = Test of Early Mathematics Ability, second edition[434]
ENCO = Emergent Numeracy and Cultural Orientations Assessment[435]
NSB = Number Sense Brief[436]

[1] RCT = Randomized controlled trial. Children, classrooms, or schools were randomly assigned to intervention conditions.

QED = Quasi-experimental design. Children, classrooms, or schools were assigned to intervention conditions by a non-random procedure.

[2] SD = Standard deviation. The information presented includes the following: (a) type of program and unit of assignment, if the study is an RCT and it differs from the unit of analysis; (b) the number of children by intervention status; and (c) the age of children in the sample.

[3] Regular classroom instruction: The researchers did not provide any additional instructional material to the comparison group. If details were available on the curriculum the comparison teachers used, it is noted parenthetically.

Treated comparison: The comparison group received additional instruction or materials from the researchers, although the topic may not have been math. If details were available on what was provided, it is noted parenthetically.

[4] All effect sizes and significance levels are calculated by the WWC unless otherwise noted. WWC calculations sometimes differ from author-reported results, due to WWC adjustments for baseline differences, clustering, or multiple comparisons. Effect sizes that were significant ($p \leq 0.05$) by WWC calculations or author calculations where no WWC adjustment was required are marked with an asterisk (*); "ns" refers to effects that were not significant. Only outcomes that met WWC evidence standards are listed here. Positive findings favor the intervention group and are either significant or substantively important (i.e., the effect size is 0.25 SD or larger). Negative findings favor the comparison group and are either significant or substantively important (i.e., the effect size is −0.25 SD or larger). "No discernible" effects are findings that are neither significant nor substantively important.

[5] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for clustering within classrooms or schools. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[6] The difference between the intervention and comparison groups was the use of the *Math Is Everywhere* activities to help teachers incorporate math in other parts of the school day, such as circle time, transitions from one activity to another, or meals.

[7] The level of statistical significance was reported by the study authors or, where necessary, calculated by the WWC to correct for multiple comparisons. For an explanation of these adjustments, see the *WWC Procedures and Standards Handbook, Version 2.1* (http://whatworks.ed.gov).

[8] In **Barnett et al. (2008)**, the difference between the intervention and comparison groups with respect to math instruction is not known. The intervention group participated in *Tools of the Mind*, a comprehensive early childhood curriculum with a math component that supported incorporating math into other parts of the school day. The comparison group participated in a district-created balanced literacy curriculum. From the information provided, it was not clear how the intervention and comparison groups differed with respect to incorporating math into other aspects of the school day.

[9] **Clements and Sarama (2007b)** also reported scores for the subscales of the Number and Geometry scales; positive effects were seen for each subscale. Findings from **Clements and Sarama (2007b)** were previously reported in the WWC intervention report on *SRA Real Math Building Blocks PreK*. The panel reports the same findings as presented in the intervention report.

[10] In **Clements and Sarama (2007b)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the curricula used in the comparison classrooms, including *Creative Curriculum*, a branded comprehensive early childhood curriculum. The intervention group participated in *Building Blocks*, a math curriculum that included regular math lessons, incorporated math into other aspects of the school day, and used games to reinforce math skills. The comparison group participated in a variety of curricula, including *Creative Curriculum*, which included regular math lessons.

[11] For **Clements and Sarama (2008)**, the WWC is reporting author-reported effect sizes consistent with prior reporting of findings from this study in the WWC intervention report on *SRA Real Math Building Blocks PreK*.

[12] **Clements et al. (2011)** also reported the subscale scores from the REMA. Findings for the subscale scores were consistent with the total score findings and were generally positive (9 of 13 scores). No discernible effects were seen for 4 of the 13 subscale scores (two in the geometry domain: transformations/turns and comparing shapes; one in the operations domain: arithmetic, and one in the basic number concepts domain: composition of number).

13 In **Clements et al. (2011)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between *Building Blocks* and the various branded curricula used in the comparison classrooms, including *DLM Early Childhood Express*. The intervention group participated in *Building Blocks*, a math curriculum that included regular math lessons, incorporated math into other aspects of the school day, and used games to reinforce math skills. The comparison group participated in a number of branded curricula, including *DLM Early Childhood Express*, an early childhood curriculum that included regular math lessons, incorporated math into other aspects of the school day, and used games to reinforce math skills.

[14] **Dyson, Jordan, and Glutting (2013)** reported total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all subscales at posttest and maintenance, except for the WJ-III–Applied Problems subscale, for which no discernible effects were seen at posttest or maintenance.

[15] In **Dyson, Jordan, and Glutting (2013)**, the difference between the intervention and comparison groups as the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, generally 3 a week, for a total of 24 sessions (or 12 hours). The sessions included instruction in number and operations in regular supplemental lessons and used games to reinforce skills, including *The Great Race*. The comparison group did not receive this additional instruction; rather, they received only the regular classroom math instruction. The regular classroom math instruction, for both the intervention and comparison children, was *Math Trailblazers*, a branded math curriculum used to teach number and operations but not guided by a developmental progression.

[16] **Fantuzzo, Gadsden, and McDermott (2011)** reported on four waves of data collection. The panel decided to use Wave 1 as pretest data, because it was collected prior to the delivery of math content. Wave 4 was used as the posttest, as it was collected at the end of the school year and delivery of the intervention. Waves 2 and 3 could be viewed as intermediary outcomes, but the panel chose to focus on posttests when determining levels of evidence.

[17] In **Fantuzzo, Gadsden, and McDermott (2011)**, the difference between the intervention and comparison groups included any aspect of instruction that differed between *EPIC* and *DLM Early Childhood Express*, a branded comprehensive early childhood curriculum. Both curricula provided regular math lessons and incorporated math into other aspects of the school day.

[18] **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)** reported other outcomes for which no pretest data were provided. The WWC was unable to conduct a review that included these outcomes, as baseline equivalence could not be established.

[19] **Jordan et al. (2012)** reported posttest and maintenance effects for total and subscale scores for the NSB, as well as the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales and a WJ-III Total, which is the sum of the WJ-III–Applied Problems and WJ-III–Calculation Problems subscales. Positive effects were found for all but seven of the NSB outcomes that were reported as no discernible effects.

[20] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The intervention group participated in additional number sense instruction that included regular math lessons and used games to reinforce math skills, including *The Great Race*. The comparison group did not receive this additional instruction in math; rather, they only had the regular classroom instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*, both of which are commercially available curricula.

[21] There were two comparisons in **Jordan et al. (2012)**. In this comparison, the difference between the intervention and comparison groups was the additional 12 hours of math instruction the intervention group received. The intervention group participated in 30-minute sessions, 3 times a week, for a total of 24 sessions (or 12 hours). The intervention group participated in additional number sense instruction that included regular math lessons and used games to reinforce math skills, including *The Great Race*. The comparison group did not receive this additional instruction in math; rather, they only had the regular classroom instruction and additional literacy instruction. The regular classroom instruction, for both the intervention and comparison children, was *Math Trailblazers* or *Math Connects*, both of which are commercially available curricula.

[22] In **Klein et al. (2008)**, the difference between the intervention and comparison groups encompassed any aspect of instruction that differed between the combined *Pre-K Mathematics* and *DLM Early Childhood Express* intervention and the curricula used in the comparison classrooms, including *Creative Curriculum*. The intervention group, which participated in a combination of *Pre-K Mathematics* and *DLM Early Childhood Express*, included regular math lessons, incorporated math into other aspects of the school day, and used games to reinforce math skills. The comparison group participated in a number of branded curricula, including *Creative Curriculum*, a comprehensive early childhood curriculum that included regular math lessons.

[23] The panel focused on the comparisons between the three intervention groups for this recommendation.

[24] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups was the manner in which number sense instruction was delivered. The intervention group participated in number sense instruction using stories to reinforce concepts and skills. The comparison group participated in the same number sense curriculum delivered in small groups, without the use of stories.

[25] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups was the manner in which number sense instruction was delivered. The intervention group participated in number sense instruction using movement to reinforce concepts and skills. The comparison group participated in the same number sense curriculum delivered in small groups, without the use of movement.

[26] There were three possible comparisons in **Monahan (2007)**. In this comparison, the difference between the intervention and comparison groups was the manner in which number sense instruction was delivered. The intervention group participated in number sense instruction using movement to reinforce concepts and skills. The comparison group participated in the same number sense instruction using stories without movement to reinforce concepts and skills.

[27] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel rated the study differently but reports the same findings as presented in the intervention report. The difference in study rating is due to the use of WWC Version 2.1 standards as opposed to WWC Version 1.0 standards. Findings from this study of *Bright Beginnings* were previously reported in the WWC intervention report on *Bright Beginnings*. The panel reports the same findings as reported in the intervention report. For both *Creative Curriculum* and *Bright Beginnings*, the authors report on additional outcomes that were assessed in the spring of kindergarten.

[28] Findings from this study of *Creative Curriculum* were previously reported in the WWC intervention report on *Creative Curriculum*. The panel reports the same findings as presented in the intervention report.

[29] Findings from these studies (**Ramani & Siegler, 2008**; **Siegler & Ramani, 2008**; **Siegler & Ramani, 2009**) were previously reported in the WWC practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*. The panel reports the findings as discussed in that practice guide.

[30] In both **Ramani and Siegler (2008)** and **Siegler and Ramani (2008)**, the difference between the intervention and comparison groups was the nature of the board games played. The intervention group played a number-based version of *The Great Race* with each space on the board having a number and children stating the number as they moved their token. The comparison group also played *The Great Race*, but with spaces that were colored and children stating the color as they moved their token.

[31] The effect is in the desired direction, with the intervention making fewer errors than the comparison group, which results in a negative effect size. However, to present the findings in a consistent manner, the effect size is reported as positive.

[32] There are two comparisons in **Ramani and Siegler (2011)**. In this comparison, the difference between the intervention and comparison groups was whether they played linear, number-based board games to reinforce math concepts and skills. The intervention group played a linear version of *The Great Race* with each space on the board having a number and children stating the number as they moved their token. The comparison group practiced counting number strings and objects and identifying numerals.

[33] There are two comparisons in **Ramani and Siegler (2011)**. In this comparison, the difference between the intervention and comparison groups was whether they played circular number-based board games to reinforce math concepts and skills. The intervention group played a circular version of *The Great Race* with each space on the board having a number and children stating the number as they moved their token. The comparison group practiced counting number strings and objects and identifying numerals.

[34] **Sarama et al. (2008)** reported subscale scores as well; however, only the means were provided, so the WWC was unable to calculate effect sizes for the subscales.

[35] The authors reported non-significant findings for these outcomes and comparisons but did not report effect sizes or provide sufficient information for the WWC to calculate effect sizes. The panel reports on these outcomes and comparisons in a manner similar to the WWC practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*.

[36] There are two comparisons in **Siegler and Ramani (2009)**. In this comparison, the difference between the intervention and comparison groups was whether they played linear number-based board games to reinforce math concepts and skills. The intervention group played *The Great Race* with each space on the board having a number and children stating the number as they moved their token. The comparison group practiced counting number strings and objects and identifying numerals.

[37] There are two comparisons in **Siegler and Ramani (2009)**. In this comparison, the difference between the intervention and comparison groups was whether they played circular number-based board games to reinforce math concepts and skills. The intervention group played *The Great Race* with each space on the board having a number and children stating the number as they moved their token. The comparison group practiced counting number strings and objects and identifying numerals.

[38] In **Sophian (2004)**, the difference between the intervention and comparison groups was whether children received math instruction using a researcher-developed, measurement-focused curriculum. The intervention group participated in a researcher-developed, measurement-focused curriculum that emphasized the concept of unit and incorporated math into other aspects of the school day. The comparison group participated in a literacy curriculum. There is no description of the math instruction children in the comparison group may have received as part of their regular classroom instruction.

Endnotes[437]

Please note that there will still be some footnotes in the guide—these will be attached to titles of the sections specifically to state that, "Eligible studies that meet WWC evidence standards or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages."

1. Following WWC guidelines, improved outcomes are indicated by either a positive statistically significant effect or a positive, substantively important effect size. The WWC defines substantively important, or large, effects on outcomes to be those with effect sizes greater than or equal to 0.25 standard deviations. See the WWC guidelines at http://whatworks.ed.gov.
2. For more information, see the WWC Frequently Asked Questions page for practice guides, http://whatworks.ed.gov.
3. This includes randomized controlled trials (RCTs) and quasi-experimental design studies (QEDs). Studies not contributing to levels of evidence include single-case designs (SCDs) evaluated with WWC pilot SCD standards and regression discontinuity designs (RDDs) evaluated with pilot RDD standards.
4. The research may include studies generally meeting WWC standards and supporting the effectiveness of a program, practice, or approach with small sample sizes and/or other conditions of implementation or analysis that limit generalizability. The research may include studies that support the generality of a relation but do not meet WWC standards; however, they have no major flaws related to internal validity other than lack of demonstrated equivalence at pretest for QEDs. QEDs without equivalence must include a pretest covariate as a statistical control for selection bias. These studies must be accompanied by at least one relevant study meeting WWC standards.
5. American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (1999).
6. Ginsburg, Klein, and Starkey (1998).
7. Underlined terms in this practice guide are defined in the Glossary.
8. Early math content areas are the specific math topics the panel believes should become the foundation of preschool, prekindergarten, and kindergarten curricula. The panel has identified number and operations, geometry, patterns, measurement, and data analysis as critical to children's math learning.
9. See, for example, **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Klein et al. (2008)**; **Sarama et al. (2008)**. Throughout this guide, eligible studies that meet WWC evidence standards or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages. See pages 11–14 for additional details on how research evidence is used in WWC practice guides.
10. See, for example, National Council of Teachers of Mathematics (2006); National Research Council (2009); National Association for the Education of Young Children (NAEYC) and National Council of Teachers of Mathematics (NCTM) (2010).
11. See, for example, **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
12. Claessens, Duncan, and Engle (2009); Claessens and Engel (2011); Duncan et al. (2007); Lee and Burkam (2002).
13. Clements and Sarama (2007); Entwisle and Alexander (1990); Ginsburg and Russell (1981); **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; Jordan, Huttenlocher, and Levine (1992); Klibanoff et al. (2006); Lee and Burkam (2002); Saxe et al. (1987); Secada (1992); Starkey, Klein, and Wakeley (2004); U.S. Department of Education, National Center for Educational Statistics (2001).
14. Aunola et al. (2004); Jordan et al. (2009); Jordan et al. (2006).
15. Stevenson et al. (1990); Gonzales et al. (2008).
16. Jordan et al. (2009); Duncan et al. (2007); Locuniak and Jordan (2008).
17. NAEYC and NCTM (2010).

18. National Governors Association Center for Best Practices, Council of Chief State School Officers (2010).
19. New York State Department of Education (2011).
20. For example, positive effects in math achievement are seen in **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Dyson, Jordan, and Glutting (2013)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**.
21. Although there is little direct evidence that identifies which specific developmental progression is most effective in teaching math to young children, the panel believes there is indirect evidence from **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; and **Fantuzzo, Gadsden, and McDermott (2011)** demonstrating developmental progressions are necessary. The panel also considered research that supports the use of the specific steps outlined in the sequence of a developmental progression, even if the developmental progression as a whole was not directly tested, for example, Purpura, Baroody, and Lonigan (in press).
22. Sarama and Clements (2009a).
23. Although unstructured or unguided learning opportunities certainly play a role in young children's mathematical learning, structured or guided opportunities also have an important role. For instance, Mix, Moore, and Holcomb (2011) found that 3-year-olds provided with toys and a matching container (e.g., wiffle balls and a muffin tin) and asked to complete a challenging equivalence (number-matching) task outperformed children who were provided with the same toys but not given a container.
24. Although instructional methods may vary across countries, the content of early math is quite similar internationally; therefore, the panel did not consider a geographic or language restriction in the review.
25. The guide focuses on teaching math to children attending preschool, prekindergarten, or kindergarten. The panel considered research examining the math competencies of infants and toddlers to identify what skills children have when they enter preschool, prekindergarten, or kindergarten. However, the literature is not included in the body of evidence as studies of infants and toddlers fall outside the age requirements of the review protocol. The panel acknowledges there is considerable debate about both the findings from research on the early math competencies of infants and toddlers and the subsequent implications on early childhood math instruction. Research informing the debate on what number and arithmetic knowledge infants and toddlers may have includes Condry and Spelke (2008); Huttenlocher, Jordan, and Levine (1994); LeCorre and Carey (2008); Wakeley, Rivera, and Langer (2000); Gelman and Butterworth (2005); Mix (2009); Sarnecka et al. (2007); Spelke (2003); and Spelke and Tsivkin (2001).
26. Reviews of studies for this practice guide applied *WWC Procedures and Standards Handbook, Version 2.1* standards. See http://whatworks.ed.gov. The protocol guiding reviews for this practice guide can be found at http://whatworks.ed.gov.
27. Table D.1 summarizes which studies are linked to which recommendations.
28. National Research Council (2009).
29. The panel acknowledges that researchers have presented different developmental progressions. For examples of curricula based on a developmental progression, see **Barnett et al. (2008)**; Clements and Sarama (2007a); **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Sarama et al. (2008)**.
30. Developmental progressions for number knowledge and other math skills can be found within curricula. For examples of articles describing such curricula, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **Preschool Curriculum Evaluation Research (PCER) Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**. These developmental progressions can also be found in other

math resources such as National Research Council (2009).

31. Jordan et al. (2006); Jordan et al. (2009); Locuniak and Jordan (2008); Palmer and Baroody (2011); Purpura, Baroody and Lonigan (in press).
32. Ginsburg, Klein, and Starkey (1998).
33. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Baroody, Eiland, and Thompson (2009)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **Lai, Baroody, and Johnson (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Ramani and Siegler (2008)**; **Sarama et al. (2008)**; **Siegler and Ramani (2008)**; **Sood (2009)**.
34. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
35. **Baroody, Eiland, and Thompson (2009)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **Siegler and Ramani (2008)**; **Sood (2009)**; **Sophian (2004).**
36. Ten studies examined interventions that used a developmental progression to guide instruction in number and operations and found positive effects: **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
37. For example, positive effects on children's math achievement were found in nine studies, in which the comparison group also received instruction in number and operations: **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Baroody, Eiland and Thompson (2009)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; and **Lai, Baroody, and Johnson (2008**). One comparison curriculum (*Creative Curriculum*) was the intervention curriculum in two studies that found no discernible effects: **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3).**
38. Eight of the 23 studies that contributed to the body of evidence for Recommendation 1 did not provide sufficient information for the panel to determine what numbers and operations instruction children in the comparison condition received. Appendix D includes additional information on the studies contributing to the body of evidence, including descriptions of the intervention and comparison group conditions.
39. Table D.1 summarizes which studies are linked to which recommendations.
40. As used in this guide, subitizing does not refer to *nonverbal subitizing*, or the ability of preverbal infants and toddlers to distinguish between small collections—a process that may or may not involve recognizing the totals or cardinal values of the collections. Nonverbal subitizing, as the term implies, does not involve labeling the total of a collection with a number word. Subitizing will be used as Kaufman et al. (1949), who coined the term, intended—immediately recognizing the cardinal value of a collection and labeling it with the appropriate number word.
41. For example evaluations of curricula that include subitizing activities, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.
42. Baroody, Li, and Lai (2008); Benoit, Lehalle, and Jouen (2004); Clements (1999); Klein and Starkey (1988); Palmer and Baroody (2011); Starkey and Cooper (1995); Wynn (1998).
43. Adapted from Baroody, Lai, and Mix (2006) and Palmer and Baroody (2011).
44. Palmer and Baroody (2011); National Research Council (2009); Mix (2008).
45. Baroody, Lai, and Mix (2006); Benoit, Lehalle, and Jouen (2004); **Dyson, Jordan, and Glutting (2013)**; Ginsburg (1977); Mix, Huttenlocher, and Levine (2002); Palmer and Baroody (2011); Sarnecka et al. (2007);

von Glasersfeld (1982); Wagner and Walters (1982); Wynn (1992).

46. Palmer and Baroody (2011).
47. Baroody, Lai, and Mix (2006); Huttenlocher, Jordan, and Levine (1994).
48. For examples of curricula that use one-to-one counting activities, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
49. Mix et al. (2012).
50. For examples of studies of curricula that use one-to-one counting, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
51. Ginsburg, Klein, and Starkey (1998).
52. Adapted from Baroody (1987).
53. Table adapted from Baroody (1987). See also **Fantuzzo, Gadsden, and McDermott (2011)** and Fuson (1988).
54. Donaldson and Balfour (1968); Weiner (1974).
55. For examples of studies of curricula that use comparisons, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
56. Sarnecka and Carey (2008); Sarnecka and Gelman (2004).
57. In addition to placing chips on a grid as shown, children can also construct and use cardinality charts using cubes or interlocking blocks.
58. Baroody and Coslick (1998); **Dyson, Jordan, and Glutting (2013)**; National Research Council (2009).
59. For examples of curricula that use numeral activities, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
60. **Aunio, Hautamaki, and Van Luit (2005)**.
61. For examples of studies of curricula that include manipulating small sets, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
62. Baroody, Lai, and Mix (2006); **Clements and Sarama (2007b)**; Kilpatrick, Swafford, and Findell (2001); Streefland (1993).
63. Huttenlocher, Jordan, and Levine (1994); Levine, Jordan, and Huttenlocher (1992).
64. Sarama and Clements (2009a).
65. Sarama and Clements (2009b).
66. NAEYC and NCTM (2010).
67. For detailed descriptions of developmental progressions, see Sarama and Clements (2009b).
68. **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
69. **Sophian (2004)**.
70. *Building Blocks*, *LOGO*, *EPIC*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*, *Creative Curriculum*, *Bright Beginnings*, *Tools of the Mind*, and two researcher-developed curricula.
71. For example, *Building Blocks* (examined in **Clements and Sarama, 2007b**; **Clements and Sarama, 2008**; **Clements et al., 2011**; **Sarama et al., 2008**) or *Pre-K Mathematics* (examined in **Klein et al., 2008**).
72. **Casey et al. (2008)**.
73. *Tools of the Mind*, *Building Blocks*, *LOGO*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express, Bright Beginnings*, and *Creative Curriculum*.
74. **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
75. **Clements and Sarama (2007b)**; **Clem-**

ents and Sarama (2008); **Clements et al. (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Weaver (1991)**.

76. **Kidd et al. (2008)**.
77. **Barnett et al. (2008)** found no discernible effects in the general numeracy domain. No discernible effects were reported in both general numeracy and geometry in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
78. *Building Blocks*, *EPIC*, *LOGO*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*, and two researcher-developed curricula.
79. Positive effects were seen in general numeracy by **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2011)**; **Sarama et al. (2008)**. Positive effects were seen in geometry by **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Weaver (1991)**. Positive effects were seen in basic number concepts by **Clements and Sarama (2007b)**; **Clements et al. (2011)**. No discernible effects were seen in operations, general numeracy, and geometry by **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
80. *Building Blocks*, *EPIC*, *Bright Beginnings*, *Creative Curriculum*, *Pre-K Mathematics Curriculum* with *Building Blocks*, and *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*.
81. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
82. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
83. Table D.1 summarizes which studies are linked to which recommendations.
84. For examples of evaluations of curricula that teach children to recognize and identify shapes, see **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**; **Weaver (1991)**.
85. Clements et al. (1999); Ho (2003); Tsamir, Tirosh, and Levenson (2008); Smith and Geller (2004).
86. For examples of evaluations of curricula that teach children to combine and separate shapes, see **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**; **Weaver (1991)**.
87. For examples of studies of curricula that teach children to manipulate shapes, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.
88. For examples of evaluations of curricula that teach children to identify patterns, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
89. For examples of evaluations of curricula that teach children to extend, create, and correct patterns, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
90. For examples of evaluations of curricula that teach children to compare objects, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**; **Weaver (1991)**.
91. For examples of evaluations of curricula that teach children to use measurement tools, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium**

(2008, Chapter 2); **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**; **Weaver (1991)**.

92. For examples of evaluations of curricula that teach children to collect and organize information, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
93. For examples of evaluations of curricula that teach children how to represent information graphically, see **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
94. National Research Council (2009).
95. Nemeth (2012); Howard et al. (2007).
96. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; National Research Council (2009); **Sarama et al. (2008)**.
97. Baroody (1987); Baroody, Tiilikainen, and Tai (2006); Clements and Sarama (2004); **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; Clements and Sarama (2009); **Clements et al. (2011)**; Dewey (1963); Ginsburg (1977); Hatano (2003); Piaget (1964); Sarama and Clements (2009a); **Sarama et al. (2008)**; Skemp (1987).
98. Baroody (1987); Dowker (2005); Jordan, Huttenlocher, and Levine (1992); Secada (1992); Starkey and Cooper (1995).
99. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**.
100. **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
101. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
102. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.
103. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**.
104. **Klein et al. (2008)**. The CMA was developed as described in Klein, Starkey, and Wakeley (2000).
105. Table D.1 summarizes which studies are linked to which recommendations.
106. For example, in **Fantuzzo, Gadsden, and McDermott (2011)**, the teachers in the comparison condition used the High/Scope Educational Research Foundation's Preschool Child Observation Record, which is a progress-monitoring tool. In other studies (e.g., **PCER Consortium, 2008, Chapter 3**), there was limited information on the comparison condition; thus, the panel is unsure whether progress monitoring occurred.
107. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008);** James (1958); Piaget (1964).
108. Baroody and Coslick (1998).
109. NAEYC (2009).
110. Gallenstein (2005).
111. Gallenstein (2005); Clements (2004).
112. NAEYC and NCTM (2010).
113. Klibanoff et al. (2006); Levine et al. (2010).
114. **Siegler (1995)**.
115. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Fuchs, L. S., Fuchs, D., and Karns (2001)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Siegler (1995)**.
116. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
117. **Klein et al. (2008)**.

118. **Fantuzzo, Gadsden, and McDermott (2011)**.
119. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.
120. Studies that found positive effects in general numeracy: **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**. Studies that found positive effects in basic number concepts: **Clements and Sarama (2007b)**; **Clements et al. (2011)**. Studies that found positive effects in geometry: **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
121. **Fuchs, L. S., Fuchs, D., and Karns (2011)**; **Siegler (1995)**.
122. Table D.1 summarizes which studies are linked to which recommendations.
123. Six of the 16 studies contributing to the body of evidence for Recommendation 4 did not provide sufficient information for the panel to determine whether the comparison group participated in instruction that included elements of Recommendation 4. Appendix D includes additional information on the studies contributing to the body of evidence, including descriptions of the intervention and comparison group conditions.
124. For examples of evaluations of curricula that begin with a child's informal and familiar math knowledge, see **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**.
125. NAEYC and NCTM (2010); NAEYC (2009).
126. For examples of evaluations of curricula that teach children to use math vocabulary, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
127. For examples of evaluations of curricula that link children's informal knowledge to formal representations, see **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**.
128. Fuson (1992).
129. For examples of evaluations of curricula that encourage conversations around math, see **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fuchs, L. S., Fuchs, D., and Karns (2001)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
130. **Siegler (1995)**.
131. Larson and Whitin (2010); NAEYC and NCTM (2010).
132. NAEYC and NCTM (2010); National Research Council (2009).
133. Adeeb, Bosnick, and Terrell (1999); May (1993).
134. Baroody and Wilkins (1999); Ernest (1986); May (1993).
135. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Sarama et al. (2008)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
136. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
137. **Arnold et al. (2002)**.
138. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.

139. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2009)**.
140. Table D.1 summarizes which studies are linked to which recommendations.
141. Table D.1 summarizes which studies are linked to which recommendations.
142. Six of the 20 studies contributing to the body of evidence for Recommendation 5 did not provide sufficient information for the panel to determine whether the comparison condition included dedicated time for math or integration of math instruction throughout the day. Appendix D includes additional information on the studies contributing to the body of evidence, including descriptions of the intervention and comparison group conditions.
143. **Clements and Sarama (2008)**.
144. For examples of evaluations of curricula that incorporate math concepts throughout the day, see **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
145. For examples of evaluations of curricula that incorporate math concepts into other parts of the day, see **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
146. Boggan, Harper, and Whitmire (2010).
147. Baroody, Purpura, and Reid (2012); Baroody, Tiilikainen, and Tai (2006); Chi (2009); Clements and Sarama (2012); Uttal, Scudder, and DeLoache (1997).
148. For examples of evaluations of curricula that use board games to practice math skills, see **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
149. Adapted from Wynroth (1986).
150. Burton (1993).
151. Kaufman et al. (1949).
152. Eligible studies that meet WWC evidence standards or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages.
153. A finding with statistical significance is a result that is not likely to be due to chance alone. For the WWC, this is defined as a finding with a significance level less than or equal to 0.05 ($p \leq 0.05$).
154. Recognizing that some studies lack the statistical power to classify practically important effects as statistically significant, the panel also accepts substantively important effects as evidence of effectiveness. Substantively important effects are defined as an effect size greater than or equal to 0.25 or less than or equal to –0.25, as measured by Hedge's *g*.
155. For multiple comparison adjustments and cluster corrections, see the *WWC Procedures and Standards Handbook, Version 2.1* at http://whatworks.ed.gov.
156. The WWC review protocol for this practice guide identified six domains that are used to group similar outcomes typically seen in effectiveness research related to teaching math to young children. Those domains are: general numeracy, basic number concepts, number recognition, operations, geometry, and patterns and classification. The guide focuses on the broader concept of early math content areas, which encompass the outcome domains.
157. There are three editions of the TEMA: TEMA (Ginsburg & Baroody, 1983); TEMA-2 (Ginsburg & Baroody, 1990); and TEMA-3 (Ginsburg & Baroody, 2003).
158. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
159. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Dyson, Jordan, and Glutting (2013)**; **Griffin, Case, and Capodilupo**

(1995) and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**.

160. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Dyson, Jordan, and Glutting (2013)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**.
161. The description of *Bright Beginnings* is based on the description provided by the WWC in the intervention report of *Bright Beginnings* (see U.S. Department of Education, June 2009). The *Bright Beginnings* curriculum was developed by Eric Smith, former superintendent of the Charlotte-Mecklenburg Schools in conjunction with district staff and local businesses.
162. The description of *SRA Real Math Building Blocks PreK* is based on the description provided by the WWC in the intervention report of *SRA Real Math Building Blocks PreK* (see U.S. Department of Education, July 2007b). The *SRA Real Math Building Blocks PreK* was developed by Drs. Douglas Clements and Julia Sarama.
163. The description of *Creative Curriculum for Preschool* is based on the description provided by the WWC in the intervention report of *Creative Curriculum for Preschool* (see U.S. Department of Education, August 2009). The *Creative Curriculum for Preschool* was developed by Diane Trister Dodge, Laura Colker, and Cate Heroman.
164. **Sophian (2004)**.
165. **Fantuzzo, Gadsden, and McDermott (2011)**.
166. **Arnold et al. (2002)**.
167. The only studies of the effectiveness of *Pre-K Mathematics* eligible for inclusion in the body of evidence were studies that examined the effectiveness of *Pre-K Mathematics* combined with another intervention—either *DLM Early Childhood Express* software or *Building Blocks*.
168. The description of *Pre-K Mathematics Curriculum* is based on the description provided by the WWC in the intervention report of *Pre-K Mathematics Curriculum* (see U.S. Department of Education, July 2007a). The *Pre-K Mathematics Curriculum* was developed by Drs. Alice Klein and Prentice Starkey with Alma Ramirez.
169. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
170. The description of *Tools of the Mind* is based on the description provided by the WWC in the intervention report of *Tools of the Mind* (see U.S. Department of Education, September 2008). The *Tools of the Mind* curriculum was developed by Deborah J. Leong and Elena Bodrova.
171. For example, **Monahan (2007)** included three intervention groups that received the same additional number and operations instruction, but through different instructional methods (small group, story reading, or movement), and a control group that received no additional instruction in number and operations. For Recommendation 1, the panel focused on the comparisons with the control group, so the difference involved receiving additional number sense content. For Recommendation 5, the panel focused on comparisons among the three intervention groups; the difference between these groups was the method of instruction, not the content of instruction.
172. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Baroody, Eiland, and Thompson (2009)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **Lai, Baroody, and Johnson (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Ramani and Siegler (2008)**; **Sarama et al. (2008)**; **Siegler and Ramani (2008)**; **Sood (2009)**.
173. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
174. **Aunio, Hautamaki, and Van Luit (2005)** was in Finland; **Lai, Baroody,**

and Johnson (2008) was in Taiwan.

175. Positive effects were seen in general numeracy in **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**. Positive effects were seen in basic number concepts in **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**; **Sood (2009)**. Positive effects were seen in operations in **Baroody, Eiland, and Thompson (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Lai, Baroody, and Johnson (2008)**; **Sood (2009)**. Positive effects were seen in number recognition in **Sood (2009)**. Positive effects were seen in patterns and classification in **Sood (2009)**. Positive effects were seen in geometry in **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**.

176. In **Curtis, Okamoto, and Weckbacher (2009)**, a negative effect was found in the basic number concepts domain: children who did not receive adult support in counting scored higher than children who did receive adult support in counting during the balance-beam task with large differences in weights. The authors note that they did not expect the intervention to favor either group for this outcome; they expected all children to pass the items, given their age and the targeted age of the items. In **Kidd et al. (2008)**, negative effects were seen in basic number concepts, operations, and patterns and classification when comparing the numeracy instruction condition with the cognitive instruction condition. The authors expected negative effects in the patterns and classification outcomes; the outcomes are more closely aligned with the cognitive instruction condition. The authors suggest children in the cognitive instruction condition also scored higher on basic number concepts and operation outcomes due to an increased ability to think abstractly although they did not receive additional instruction in numeracy. Their increased ability to think abstractly may have enabled them to learn more from the regular classroom math instruction, which all children received. The ability to think more abstractly, and learn more from the regular classroom instruction, may be the reason for children in the cognitive instruction condition scoring higher on the basic number concepts and operations outcomes than children who received supplemental instruction in numeracy.

177. **Barnett et al. (2008)**; **Baroody, Eiland, and Thompson (2009)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Kidd et al. (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sood (2009)**.

178. **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.

179. Table D.1 summarizes which studies are linked to which recommendations.

180. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.

181. **Baroody, Eiland, and Thompson (2009)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Lai, Baroody, and Johnson (2008)**.

182. **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Lai, Baroody, and Johnson (2008)**; **Monahan (2007)**; **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**.

183. Positive effects were found in the domains

of general numeracy (**Dyson, Jordan, & Glutting, 2013**; **Jordan et al., 2012**; **Monahan, 2007**), basic number concepts (**Curtis, Okamoto, & Weckbacher, 2009**; **Ramani & Siegler, 2008**; **Siegler & Ramani, 2008**), operations (**Dyson, Jordan, & Glutting, 2013**; **Jordan et al., 2012**; **Kidd et al., 2008**; **Lai, Baroody, & Johnson, 2008**), number recognition (**Ramani & Siegler, 2008**), and patterns and classification (**Kidd et al., 2008**).

184. No discernible effects were found in the domains of operations (**Dyson, Jordan, & Glutting, 2013**; **Monahan, 2007**), basic number concepts (**Curtis, Okamoto, & Weckbacher, 2009**; **Kidd et al., 2008**), and patterns and classification (**Kidd et al., 2008**).

185. Negative effects were found in the domain of basic number concepts, which the authors noted they did not expect the intervention to favor (**Curtis, Okamoto, and Weckbacher, 2009**). **Kidd et al. (2008)** also found negative effects in basic number concepts, operations, and patterns and classification; the negative effects in basic number concepts and operations were hypothesized to be due to the comparison condition's (cognitive instruction) increased ability to think abstractly, which enabled them to learn more from the regular classroom instruction.

186. **Baroody, Eiland, and Thompson (2009)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**. In four of the five studies, the comparison group participated in a curriculum that was also of interest to the panel as it included components of this recommendation (**Clements and Sarama, 2007b**; **Clements and Sarama, 2008**; **Clements et al., 2011**; **Fantuzzo, Gadsden, and McDermott, 2011**; **Klein et al., 2008)**.

187. Positive effects were found in the domains of general numeracy (**Clements et al., 2011**; **Klein et al., 2008**; **Fantuzzo, Gadsden, & McDermott, 2011**), basic number concepts (**Clements & Sarama, 2007b**; **Clements et al., 2011**), and geometry (**Clements & Sarama, 2007b**; **Clements et al., 2011**). One study (**Baroody, Eiland, & Thompson, 2009**) found both positive and no discernible effects in the operations domain.

188. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Clements and Sarama (2008)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sood (2009)**; **Sophian (2004)**.

189. Positive effects were found in the domains of general numeracy (**Arnold et al., 2002**; **Clements & Sarama, 2008**; **Sarama et al., 2008**; **Sophian, 2004**), basic number concepts (**Aunio, Hautamaki, & Van Luit, 2005**; **Griffin, Case, & Capodilupo, 1995** and related publication **Griffin, Case, & Siegler, 1994**; **Sood, 2009**), number recognition (**Sood, 2009**), operations (**Sood, 2009**), geometry (**Aunio, Hautamaki, & Van Luit, 2005**), and patterns and classification (**Sood, 2009**).

190. No discernible effects were found in the domains of general numeracy (**PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**; **Sood, 2009**), operations (**Barnett et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**) and geometry (**Aunio, Hautamaki, & Van Luit, 2005**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**).

191. For example, in **Clements and Sarama (2007)**, the intervention, *Building Blocks*, included targeted instruction in number and operations based on learning trajectories, whereas the comparison curricula included *Creative Curriculum*, which also involved targeted instruction in number and operations based on a developmental progression. Studies in which *Creative Curriculum* was the intervention curriculum found no discernible effects (**PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**).

192. The panel reviewed studies of a supplemental number sense curriculum that provided additional targeted instruction in number and operations based on a developmental progression (**Dyson, Jordan, & Glutting, 2013**; **Jordan et al., 2012**). These studies found positive effects in the general numeracy and operations domains

at posttest and maintenance and no discernible effects in the operations domain at maintenance.

193. The interventions are as follows: *Building Blocks* (**Clements & Sarama, 2007b**; **Clements & Sarama, 2008**; **Clements et al., 2011**; **Sarama, 2008**), *Math Is Everywhere* (**Arnold et al., 2002**), the *Pre-K Mathematics Curriculum* (**Klein et al., 2008**), and *Rightstart* (**Griffin, Case, & Capodilupo, 1995** and related publication **Griffin, Case, & Siegler, 1994**).
194. *Building Blocks*, *Pre-K Mathematics Curriculum*, and *Rightstart*.
195. **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Klein et al. (2008)**.
196. **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sarama et al. (2008)**.
197. National Research Council (2009).
198. Positive effects were found in basic number concepts in the following: **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**. Positive effects were found in general numeracy in the following: **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
199. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**.
200. **Sarama et al. (2008)**.
201. **Clements and Sarama (2007b)** compared *Building Blocks* classrooms with comparison classrooms, including some that used *Creative Curriculum*, a comprehensive early childhood curriculum that includes units on number and operations based on a developmental progression. **Clements and Sarama (2008)** compared *Building Blocks* classrooms to locally developed curricula, which is likely to have included instruction in number and operations. **Clements et al. (2011)** compared *Building Blocks* classrooms with classrooms using a variety of curricula, including *DLM Early Childhood Express*, which was also developed by Clements and Sarama and includes many of the same key elements but does not use the learning trajectories (developmental progression) in the same manner as *Building Blocks*.
202. **Sarama et al. (2008)**.
203. **Klein et al. (2008)**. Based on conversations with the primary author of both *Building Blocks* and *DLM Early Childhood Express*, the panel understands that although *DLM Early Childhood Express* could be considered a precursor to *Building Blocks*, the interventions differ in some key areas—including the articulation of a learning trajectory. Thus, the panel considers the intervention tested in **Sarama et al. (2008)** to be distinct from the intervention tested in **Klein et al. (2008)**.
204. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
205. **Arnold et al. (2002)**.
206. Clements and Sarama (2007c).
207. Clements, Sarama, and Liu (2008).
208. Klein, Starkey, and Wakeley (2000).
209. Ginsburg and Baroody (1990).
210. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
211. **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
212. **Barnett et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
213. **Aunio, Hautamaki, and Van Luit (2005)**.
214. **Fantuzzo, Gadsden, and McDermott (2011)**.
215. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
216. **Barnett et al. (2008)**.
217. **Aunio, Hautamaki, and Van Luit (2005)**.
218. **Fantuzzo, Gadsden, and McDermott (2011)**. The particular assessment used was the Learning Express Mathematics Scale (McDermott et al., 2009).
219. Van Luit, Van de Rijt, and Aunio (2003).
220. Shayer and Wylam (1978).
221. Klein and Starkey (2002).
222. Woodcock, McGrew, and Mather (2007).
223. McDermott et al. (2009).

224. **Baroody, Eiland, and Thompson (2009)**; **Curtis, Okamoto, and Weckbacher (2009)**; **Dyson, Jordan, and Glutting (2013)**; **Lai, Baroody, and Johnson (2008)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Monahan (2007)**; **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**; **Sood (2009)**; **Sophian (2004)**.
225. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Kidd et al. (2008)**; **Monahan (2007)**; **Sood (2009)**; **Sophian (2004)**.
226. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**.
227. At maintenance (six weeks after the intervention), counting skills of the children who participated in the intervention were no longer significantly different from the control group, as measured by the Number Sense Brief (NSB; Jordan et al., 2010) and the Woodcock-Johnson, third edition (WJ-III; Woodcock, McGrew, & Mather, 2007). All other effects were maintained.
228. **Sood (2009)**.
229. **Sood (2009)**. At the three-week follow-up, effects were maintained in number relationships, five-and-ten-frame identification and representation, five-and-ten-frame calculation, and nonverbal calculations. Effects were not maintained for "counting from" and number identification.
230. **Monahan (2007)**.
231. **Sophian (2004)**.
232. **Kidd et al (2008)**.
233. **Curtis, Okamoto, and Weckbacher (2009)**.
234. **Curtis, Okamoto, and Weckbacher (2009)** report a negative effect in the basic number concepts domain, with children who did not receive adult support in counting scoring higher than children who did receive adult support in counting during the balance-beam task with large differences in weights. The authors note that they did not expect the intervention to favor either group for this outcome; they expected all children to pass the items, given their age and the targeted age of the items.
235. **Baroody, Eiland, and Thompson (2009)**.
236. **Lai, Baroody, and Johnson (2008)**.
237. **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**.
238. **Ramani and Siegler (2008)**.
239. **Siegler and Ramani (2008)**.
240. **Monahan (2007)**.
241. Jordan et al. (2010).
242. Pearson (n.d.).
243. Woodcock, McGrew, and Mather (2007).
244. CTB/McGraw Hill (1990).
245. **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Kidd et al. (2008)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
246. **Sophian (2004)**.
247. Positive findings were found in the following domains: geometry (**Casey et al., 2008**; **Clements & Sarama, 2007b**; **Clements et al., 2011**; **Weaver, 1991**), general numeracy (**Clements et al., 2011**; **Clements & Sarama, 2008**; **Fantuzzo, Gadsden, & McDermott, 2011**; **Klein et al., 2008**; **Sarama et al,, 2008**; **Sophian, 2004**), basic number concepts (**Clements & Sarama, 2007b**; **Clements et al., 2011**; **Kidd et al., 2008**), operations (**Kidd et al., 2008**), and patterns and classification (**Kidd et al., 2008**).
248. No discernible effects were found in the domains of geometry (**Casey et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**), operations (**Barnett et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**), and general numeracy (**PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**).
249. **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
250. *Building Blocks*, *LOGO*, *EPIC*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*, *Creative Curriculum*, *Bright Beginnings*, *Tools of the Mind*, and two researcher-developed curricula.

251. *Tools of the Mind*, *Building Blocks*, *LOGO*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express, Bright Beginnings*, and *Creative Curriculum*.
252. *Building Blocks*, *EPIC*, *LOGO*, *Pre-K Mathematics Curriculum* with *Building Blocks*, *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*, and two researcher-developed curricula.
253. *Building Blocks*, *EPIC*, *Bright Beginnings*, *Creative Curriculum*, *Pre-K Mathematics Curriculum* with *Building Blocks*, and *Pre-K Mathematics Curriculum* with *DLM Early Childhood Express*.
254. Table D.1 summarizes which studies are linked to which recommendations.
255. **Weaver (1991)** offered supplemental instruction in geometry, patterns, and measurement and found positive effects in the geometry domain. **Sophian (2004**) offered targeted instruction in geometry and measurement, while the comparison group received a literacy intervention; positive effects were found in the domain of general numeracy. **Kidd et al. (2008)** offered targeted instruction in oddity, seriation and conservation, while the comparison group received either an art intervention or a numeracy intervention; positive effects were found in the domains of basic number concepts, operations, and patterns and classification.
256. **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
257. Positive effects were found in the domains of geometry (**Casey et al., 2008**) and general numeracy (**Clements & Sarama, 2008**; **Sarama et al., 2008**).
258. No discernible effects were found in the domains of geometry (**Casey et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**), operations, (**Barnett et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**), and general numeracy (**PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**).
259. **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**.
260. **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
261. **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**.
262. **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
263. **Clements and Sarama (2008)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**.
264. Both positive effects (**Casey et al., 2008**; **Clements & Sarama, 2007b**; **Clements & Sarama, 2008**; **Weaver, 1991**) and no discernible effects (**Casey et al., 2008**; **PCER Consortium, 2008, Chapter 2**; **PCER Consortium, 2008, Chapter 3**) were found in geometry. The panel reports scale scores from **Clements et al. (2011)**; however, subscales were reported and include positive effects for three of the five geometry subscales, no discernible effects for two of the five geometry subscales, and positive effects for outcomes assessing measurement and patterns and classification. Positive effects were found in patterns and classification (**Kidd et al., 2008**).
265. **Barnett et al. (2008)**; **Casey et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**; **Weaver (1991)**.
266. **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
267. **Casey et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **Sophian (2004)**; **Weaver (1991)**.
268. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**.
269. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**.
270. **Sarama et al. (2008)**.

271. **Klein et al. (2008)**.
272. The geometry scale of the Building Blocks Assessment (Clements & Sarama, 2007c).
273. The total score on the Research-Based Early Math Assessment (REMA; Clements, Sarama, & Liu, 2008) or the Child Math Assessment (CMA; Klein, Starkey, & Wakeley, 2000).
274. **Weaver (1991)**. The publication also assessed the impact of computer-based *LOGO* compared with floor-based *LOGO* for preschool children. This contrast is not evidence for this recommendation, as both groups of children used *LOGO*; the study found no discernible effects for computer-based *LOGO*.
275. **Fantuzzo, Gadsden, and McDermott (2011)**.
276. **Sophian (2004)**.
277. **Casey et al. (2008)**.
278. **Casey et al. (2008)** also reported no discernible effects and a single negative effect in the study. The negative finding was for a new assessment of mental rotation ability. The regular classroom instruction group scored higher on the assessment at pretest and posttest. The authors suggested that the intervention experience, which involved mentally rotating individual blocks, may not have transferred to the mental rotation of more complex figures—the focus of the assessment.
279. **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
280. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Weaver (1991)** found positive effects in the domains of general numeracy and geometry. **Kidd et al. (2008)** found positive effects in basic number concepts, operations, and patterns and classification. **Barnett et al. (2008)** found no discernible effects in the operations domain. No discernible effects were reported in general numeracy, operations, and geometry for **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
281. **Clements et al. (2011)**. The development of the Research-Based Early Math Assessment (REMA) is discussed in Clements, Sarama, and Liu (2008).
282. *Building Blocks* was the focal intervention in **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**. *DLM Early Childhood Express* was combined with the *Pre-K Mathematics Curriculum* in **Klein et al. (2008)**. *Building Blocks* was combined with the *Pre-K Mathematics Curriculum* in **Sarama et al. (2008)**.
283. **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Weaver (1991)**.
284. *Building Blocks* was examined in **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**. *Bright Beginnings* and *Creative Curriculum* were assessed in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**. *EPIC* was examined in **Fantuzzo, Gadsden, and McDermott (2011)**. *Pre-K Mathematics Curriculum* was studied in **Klein et al. (2011)**. Two researcher-developed curricula were examined in **Sophian (2004)**; **Weaver (1991)**.
285. *Building Blocks*, *Bright Beginnings*, *Creative Curriculum*, *EPIC*, and the *Pre-K Mathematics Curriculum*.
286. *Building Blocks*, *Creative Curriculum*, and *EPIC*.
287. Positive effects were seen in general numeracy by **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2011)**; **Sarama et al. (2008)**. Positive effects were seen in geometry by **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Weaver (1991)**. Positive effects were seen in basic number concepts by **Clements and Sarama (2007b)**; **Clements et al. (2011)**. No discernible effects were seen in general numeracy, operations, and geometry by **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
288. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et**

al. (2011). **Clements et al. (2011)** is the only study that also reported a measurement outcome—the measurement subscale of the Early Mathematics Assessment (Clements, Sarama, & Liu, 2008)—on which children who participated in *Building Blocks* scored higher on the measurement subscale than children who participated in regular classroom instruction.

289. **Sarama et al. (2008)**.
290. **Klein et al. (2008)**.
291. **Weaver (1991)**.
292. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
293. **Sophian (2004)**.
294. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
295. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
296. **Fantuzzo, Gadsden, and McDermott (2011)**.
297. Clements and Sarama (2007c).
298. Klein, Starkey, and Wakeley (2000).
299. Klein and Starkey (2002).
300. CTB/McGraw Hill (1990).
301. McDermott et al. (2009).
302. Pearson (n.d.).
303. Clements, Sarama, and Liu (2008).
304. Wechsler (2003).
305. Woodcock and Johnson (1990).
306. Woodcock, McGrew, and Mather (2007).
307. **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
308. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
309. **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**.
310. **Dyson, Jordan, and Glutting (2013)**.
311. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
312. **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**.
313. **Dyson, Jordan, and Glutting (2013)** found positive effects in the general numeracy and operations outcome domain; however, no discernible effects were found on an operations outcome measured six weeks after the initial posttest. **Jordan et al. (2012)** found positive effects in the general numeracy and operations outcome domains; this included outcomes measured eight weeks after the initial posttest.
314. **Clements and Sarama (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
315. **Clements and Sarama (2008)**; **Sarama et al. (2008)**.
316. **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
317. **Arnold et al. (2002)**.
318. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
319. **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**.
320. **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**.
321. **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
322. **Ibid**.
323. Table D.1 summarizes which studies are linked to which recommendations.
324. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.

325. For example, in **Fantuzzo, Gadsden, and McDermott (2011)**, the teachers in the comparison condition used the High/Scope Educational Research Foundation's Preschool Child Observation Record, which is a progress-monitoring tool. In other studies, there was limited information on the comparison condition; thus, the panel is unsure whether progress monitoring occurred (e.g., **PCER Consortium, 2008, Chapter 3**).
326. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Sarama et al. (2008)**.
327. **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
328. **Clements and Sarama (2007b)**.
329. **Clements et al. (2011)**.
330. **Clements and Sarama (2008)**.
331. Clements, Sarama, and Liu (2008).
332. **Sarama et al. (2008)**.
333. **Klein et al. (2008)**.
334. The CMA was developed as described in Klein, Starkey, and Wakeley (2000).
335. **PCER Consortium (2008, Chapter 2)**.
336. **PCER Consortium (2008, Chapter 3)**.
337. **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**.
338. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**.
339. **Fantuzzo, Gadsden, and McDermott (2011)**.
340. **Arnold et al. (2002)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
341. **Arnold et al. (2002)**.
342. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
343. Clements and Sarama (2007c).
344. Clements, Sarama, and Liu (2008).
345. Klein, Starkey, and Wakeley (2000).
346. Klein and Starkey (2002).
347. Jordan et al. (2010).
348. McDermott et al. (2009).
349. Woodcock, McGrew, and Mather (2007).
350. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Fuchs, L. S., Fuchs, D., and Karns (2001)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Siegler (1995)**.
351. Quasi-experimental designs: **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
352. Positive effects in general numeracy were found in **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**. Positive effects in operations were found in **Jordan et al. (2012)**. Both positive and no discernible effects in operations were found in **Dyson, Jordan, and Glutting (2013)**. Both positive and no discernible effects in general numeracy were found in **Fuchs, L. S., Fuchs, D., and Karns (2001)**. No discernible effects in general numeracy, operations, and geometry were reported in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
353. Positive effects in geometry were found in **Clements and Sarama (2007b)**; **Clements et al. (2011)**. No discernible effects in geometry were reported in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
354. **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
355. **Siegler (1995)**. The negative finding was for the comparison between children who received feedback and then had to explain their own reasoning and children who received feedback only. The authors noted that explanations may enhance learning, particularly when a correct response is explained.
356. Table D.1 summarizes which studies are linked to which recommendations.
357. **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott**

(2011); **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.

358. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**.
359. The scope of this practice guide is limited to strategies to increase math communication in classrooms. For a wider review of strategies to ask deep explanatory questions in classrooms, see Pashler et al. (2007), specifically Recommendation 7.
360. **Dyson, Jordan, and Glutting (2013)**; **Fuchs, L. S., Fuchs, D., and Karns (2001)**; **Jordan et al. (2012)**; **Siegler (1995)**.
361. **Fuchs, L. S., Fuchs, D., and Karns (2001)**; **Jordan et al. (2012)**.
362. **Dyson, Jordan, and Glutting (2013)**.
363. **Siegler (1995)**. The negative finding was for the comparison between children who received feedback and then had to explain their own reasoning and children who received feedback only. The authors noted that explanations may enhance learning, particularly when a correct response is explained.
364. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**.
365. Positive effects in the outcome domain of general numeracy were reported in **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**. No discernible effects in general numeracy were reported in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
366. Positive effects in geometry were found in **Clements and Sarama (2007b)**. No discernible effects in geometry were reported in **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
367. **Clements and Sarama (2007b)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
368. **Barnett et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
369. **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Klein et al. (2008)**.
370. **Ibid**.
371. **Clements et al. (2011)**.
372. **Ibid**.
373. *Bright Beginnings*, *Building Blocks*, *Creative Curriculum, EPIC*, *Pre-K Mathematics Curriculum*, *Tools of the Mind*, and a researcher-developed number sense curriculum.
374. **Fuchs, L. S., Fuchs, D., and Karns (2011)**; **Siegler (1995)**.
375. **Fuchs, L. S., Fuchs, D., and Karns (2011)**.
376. See Madden, Gardner, and Collins (1987) for additional information on the SESAT.
377. See Gardner et al. (1987) for additional information on the SAT-P.
378. **Siegler (1995)**.
379. **Arnold et al. (2002)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Sarama et al. (2008)**; **Sophian (2004)**.
380. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**. Two studies assessed the effects of *Building Blocks* combined with *Pre-K Mathematics* (**Sarama et al., 2008**) or *DLM Early Childhood Express*, a curriculum related to *Building Blocks*, and *Pre-K Mathematics* (**Klein et al., 2008**).
381. **Fantuzzo, Gadsden, and McDermott**

(2011).

382. Clements and Sarama (2007c).
383. Klein, Starkey, and Wakeley (2000).
384. Klein and Starkey (2002).
385. Madden, Gardner, and Collins (1987).
386. Gardner et al. (1987).
387. Jordan et al. (2010).
388. Clements, Sarama, and Liu (2008).
389. Woodcock and Johnson (1990).
390. Woodcock, McGrew, and Mather (2007).
391. CTB/McGraw Hill (1990).
392. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Sarama et al. (2008)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
393. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Sophian (2004)**.
394. **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Ramani and Siegler (2008)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
395. Positive effects: **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**. No discernible effects: **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
396. Positive effects: **Ramani and Siegler (2008)**. No discernible effects: **Ramani and Siegler (2011)**; **Siegler and Ramani (2009)**.
397. Positive effects: **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2011)**. No discernible effects: **Barnett, et al. (2008)**; **Dyson, Jordan, and Glutting (2013)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2009)**.
398. Positive effects: **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements et al. (2011)**. No discernible effects: **Aunio, Hautamaki, and Van Luit (2005)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
399. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Dyson, Jordan, and Glutting (2013)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Jordan et al. (2012)**; **Klein et al. (2008)**; **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**.
400. **Arnold et al. (2002)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
401. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
402. Table D.1 summarizes which studies are linked to which recommendations.
403. National Research Council (2009).
404. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Monahan (2007)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
405. Positive effects in general numeracy: **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Monahan (2007)**. No discernible effects in general numeracy: **Monahan (2007)**. Positive effects in basic number concepts: **Ramani and Siegler (2008)**; **Siegler and Ramani**

(2008); **Siegler and Ramani (2009)**. No discernible effects in basic number concepts: **Siegler and Ramani (2009)**. Positive effects in number recognition: **Ramani and Siegler (2008)**. No discernible effects in number recognition: **Ramani and Siegler (2011)**; **Siegler and Ramani (2009)**. Positive effects in operations: **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2011)**. No discernible effects in operations: **Dyson, Jordan, and Glutting (2013)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.

406. **Arnold et al. (2002)**; **Aunio, Hautamaki, and Van Luit (2005)**; **Barnett et al. (2008)**; **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**; **Klein et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**; **Sarama et al. (2008)**; **Sophian (2004)**.
407. **Arnold et al. (2002)**; **Clements and Sarama (2008)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
408. **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**; **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
409. **Aunio, Hautamaki, and Van Luit (2005)**; **Clements and Sarama (2007b)**.
410. **Monahan (2007)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
411. **Barnett et al. (2008)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
412. **Aunio, Hautamaki, and Van Luit (2005)**; **PCER Consortium (2008, Chapter 2)**; **PCER Consortium (2008, Chapter 3)**.
413. **Clements et al. (2011)**; **Fantuzzo, Gadsden, and McDermott (2011)**.
414. **Ibid**.
415. **Clements et al. (2011)**.
416. **Ibid**.
417. **Arnold et al. (2002)**.
418. **Clements and Sarama (2007b)**; **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
419. **Monahan (2007)**; **Sophian (2004)**.
420. **Clements and Sarama (2008)**; **Clements et al. (2011)**; **Klein et al. (2008)**; **Monahan (2007)**; **Sarama et al. (2008)**; **Sophian (2004)**.
421. **Clements and Sarama (2007b)**; **Clements et al. (2011)**.
422. **Ibid**.
423. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**; **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Sarama et al. (2008)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**; **Sophian (2004)**.
424. **Ramani and Siegler (2008)**; **Ramani and Siegler (2011)**; **Siegler and Ramani (2008)**; **Siegler and Ramani (2009)**.
425. **Dyson, Jordan, and Glutting (2013)**; **Jordan et al. (2012)**.
426. Clements and Sarama (2007c).
427. Clements, Sarama, and Liu (2008).
428. **Griffin, Case, and Capodilupo (1995)** and related publication **Griffin, Case, and Siegler (1994)**.
429. CTB/McGraw Hill (1990).
430. McDermott et al. (2009).
431. Woodcock and Johnson (1990).
432. Woodcock, McGrew, and Mather (2007).
433. Klein, Starkey, and Wakeley (2000).
434. Ginsburg and Baroody (1990).
435. **Monahan (2007)**.
436. Jordan et al. (2010).
437. Eligible studies that meet WWC evidence standards or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages. For more information about these studies, please see Appendix D.

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