Schema Based Instruction

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Please work on the following...

If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?
Variations of Set Up

\[
\frac{300 \text{ cal}}{x \text{ cal}} = \frac{100 \text{ g}}{30 \text{ g}} \quad \text{and} \quad \frac{300 \text{ cal}}{100 \text{ g}} = \frac{x \text{ cal}}{30 \text{ g}}
\]

\[
\frac{x \text{ cal}}{300 \text{ cal}} = \frac{30 \text{ g}}{100 \text{ g}} \quad \text{and} \quad \frac{100 \text{ g}}{300 \text{ cal}} = \frac{30 \text{ g}}{x \text{ cal}}
\]
Original Problem:
If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

Variations:

a) How many calories are in 30 g of a certain food, given that there are 300 calories in 100 g of the same food?
b) A serving size of 100 g of a certain food has 300 calories. How many calories would a smaller serving size of 30 g have?
c) Your dad gives you a 30 ounce Hershey’s dark chocolate mega-kiss in your lunch box. If a 100 ounce brick of Hershey’s chocolate contains 300 micropops of caffeine, how much caffeine is in the mega-kiss?
What’s the Deal?

What makes it difficult?

Students successful with word problems...
What’s the Key

• Understand the Problem
• Construct an accurate representation
• Generate, Plan, and Monitor the solution
• Execute the computation
• Meaningfully interpret the solution
What’s the Key?

Students successful with word problems... distinguish “relevant information from irrelevant, perceiving rapidly and accurately the mathematical structure of the problems and generalizing across a wider range of mathematically similar problems.”

(Van Dooren, 2010)
What’s the Key?

Students successful with word problems...

• Relevant vs. Irrelevant

• Structure across mathematically similar problems

(Van Dooren, 2010)
Key Words

**CUBES**
- **C**omprehend
- **U**nderstand
- **B**uild
- **E**valuate
- **S**ynthesize

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**CONQUER THE PROBLEM!!!**

**BEFORE**
- PLAN
  - *Read & visualize*
  - *Reread & code*
  - *Sketch & predict*
- **What is the problem asking?**
- **What would be a reasonable answer?**

**DURING**
- **SOLVE**
  - *Show my strategies*
  - *Show my thinking*
- **Are my strategies effective and efficient?**
- **Is there another way to solve?**

**AFTER**
- **CHECK**
  - *Check my work*
  - *Go back to the question*
  - *Answer in a complete sentence*
  - *Did I answer the question?*
  - *Does my answer make sense?*

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www.ksdetasn.org/mtss
Traditional Approaches

• Key Words
• Draw a picture
• Heuristic
• Domain Specific Practice
Traditional Approaches

• Key Word Approach
  – Does not develop sense making
  – Lacks the structure to expand to more complicated problems
  – Words appear too often
  – Multi-Step problems (which begin in 2nd grade)

(Karp, 2017)
Draw a Picture

There are 4 adults and 2 children who need to cross the river. A small boat is available that can hold either 1 adult or 1 or 2 small children. Everyone can row the boat. How many one-way trips does it take for all of them to cross the river?
Draw a Picture

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Schema’s Visual Representation

- Organize and summarize the info
- Make connections to the concrete
- Reasoning the story’s situations

- All create reduction in working memory
- When attached to a schema, transfer becomes strong

(Jitendra, 2013)
Schema vs Draw a Picture

• What is the difference?
Schema vs Draw a Picture

(Jitendra, 2016)
Schematic Diagrams

• Highlight the relationships within the problem
• Establish deep understanding of the relationships
• Problems can be categorized by the student who understands these relationships
  – ...“generalizing across a wider range of mathematically similar problems.”
SBI (Schematic Based Instruction)

- Problem Schema Identification
- Representation
- Planning
- Solution
Conceptual Knowledge

• Problem Schema Identification
• Representation
• Planning
• Solution

• Schema Knowledge
• Elaboration Knowledge
• Strategic Knowledge
• Execution Knowledge

(Jitendra, 2013)

(Marshall, 1995)
Problem Schema Identification

• Organized structure of given elements and relations specific to a situation
• Bring clarity to the links of both the relationships and patterns of given operations

(Jitendra, 2013)
Problem Schema Identification

• What type of structure exists in the problem?
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<td>Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 − 3 = ?</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2</td>
</tr>
<tr>
<td><strong>Compare</strong></td>
<td>(&quot;How many more?&quot; version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 − 2 = ?</td>
<td>(&quot;How many more?&quot; version): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?</td>
<td>(&quot;Version with “more”&quot;): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? How many apples does Lucy have? 5 − 3 = ?, ? + 3 = 5</td>
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Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and

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| $3 + 2 = ?$ | $3 + ? = 5, \quad 5 - 3 = ?$ | $5 = 0 + 5, \quad 5 = 5 + 0$
$5 = 1 + 4, \quad 5 = 4 + 1$
$5 = 2 + 3, \quad 5 = 3 + 2$ |

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Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and...
<table>
<thead>
<tr>
<th>Unknown Product</th>
<th>Group Size Unknown (&quot;How many in each group?&quot; Division)</th>
<th>Number of Groups Unknown (&quot;How many groups?&quot; Division)</th>
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<tbody>
<tr>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18, \quad 18 \div 3 = ?$</td>
<td>$? \times 6 = 18, \quad 18 \div 6 = ?$</td>
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**Equal Groups**

- **There are 3 bags with 6 plums in each bag. How many plums are there in all?**
  - *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?

- **If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?**
  - *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?

- **If 18 plums are to be packed 6 to a bag, then how many bags are needed?**
  - *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?

**Arrays, Area**

- **There are 3 rows of apples with 6 apples in each row. How many apples are there?**
  - *Area example.* What is the area of a 3 cm by 6 cm rectangle?

- **If 18 apples are arranged into 3 equal rows, how many apples will be in each row?**
  - *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?

- **If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?**
  - *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

**Compare**

- **A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?**
  - *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?

- **A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?**
  - *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?

- **A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?**
  - *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

**General**

- $a \times b = ?$
- $a \times ? = p$ and $p + a = ?$
- $? \times b = p$ and $p + b = ?$
Problem Schema Identification

• What type of structure exists in the problem?
  – Ratios
  – Proportions
  – Percents
Representation

- Explicitly Taught ("Story Situation")
- Avoids irrelevant information
- Formats the problem
- Connects to the computation
- Vertically aligned

(Jitendra, 2013)
**Change Story Situation**
John had 47 baseball cards in his collection. He lost 15 of them when his family moved from Florida to New York. Now John has 32 baseball cards.

![Diagram of change set](Image)

(Jitendra, 2002)
Story Situation

**Group Story Situation**
Tim has 54 fruit trees in his orchard. 39 are apple trees, and the remaining 15 are peach trees.

(Jitendra, 2002)
**Compare Story Situation**

Mitch has 43 CDs and Anne has 70. Anne has 27 more CDs than Mitch.

- **Anne**: 70 CDs
- **Mitch**: 43 CDs
- **Difference set**: 27 CDs
- **Compared set**: 70 CDs
- **Referent set**: 43 CDs

(Jitendra, 2002)
**Change Problem**

A balloon man had some balloons. Then 14 balloons blew away and the man now has 29 balloons. How many balloons did the man begin with?

**Change set**

14 balloons

? balloons

Beginning set

T

29 balloons

Ending set

Total is not known, so add.

29 + 14 = 43

The man began with 43 balloons.

(Jitendra, 2002)
**Group Problem**

Jenny saw 25 birds on a camping trip. She saw 17 sparrows and some owls. How many owls did Jenny see on the camping trip?

```
17  ?
```

Total is known, so subtract.
```
25 - 17 = 8
Jenny saw 8 owls on the camping trip.
```
Compare Problem

Barbara is 37 years old. Cindy is 7 years older than Barbara. How old is Cindy?

Total is not known, so add.

37 + 7 = 44
Cindy is 44 years old.

(Jitendra, 2002)
Planning

• Self-Monitor from start
  – Think about the needed solution
    • Estimate the solution (not too exact)
    • Rewrite the question
  – Determine Structure and Model
  – Plan the computation strategy (ie – Subtract the lesser, cross-multiply, equivalent fractions, etc.)
  – Solve the Computation

(Jitendra, 2013)
Solution

• What does the computation mean?
• How close is my estimate?
• Does my answer make sense?
• Does it answer the question being asked?
Try to find Varying Diagrams to Explore and Discuss

• Examples and links from Free resources
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| 5 − 2 = ?       | 5 − ? = 3                                                                       | ? − 2 = 3                                                                       |

(Change set)

14 balloons

? balloons

29 balloons

Beginning set

Ending set

(Jitendra, 2002)
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(Jitendra, 2002)

(Sparrows 17, Owls ?) → (Smaller sets T) → (Larger set T) → (Birds 25)

(T)

(A₁)

(A₂)

(KSDE, 2018)
### Compare

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### Compare Problem

Barbara is 37 years old. Cindy is 7 years older than Barbara. How old is Cindy?

Cindy

- 7 years

Barbara

- 37 years

Compared set T

Diff erence set

Referent set

Total is not known, so add.

37 + 7 = 44

Cindy is 44 years old.

(KSDE, 2018)
Ratio Problems

Problem 2: There are 28 employees at the local bank. Every morning 5 out of 7 employees use Route A to drive to work. How many employees use Route A to drive to work? (Problem 4.3)

\[
\frac{x}{28} = \frac{5}{7}
\]

(Jitendra, 2016)
Proportion Problem

Carlos is on the school’s track team. He takes 54 minutes to run 6 miles. Assuming that he runs at a constant pace for all 6 miles, how long did it take him to run 2 miles? (Problem 7.2)
Proportion Problem

Carlos is on the school’s track team. He takes 54 minutes to run 6 miles. Assuming that he runs at a constant pace for all 6 miles, how long did it take him to run 2 miles? (Problem 7.2)

If 54 minutes Then 6 miles

If 6 miles Then 2 miles

If 54 minutes Then 2 miles

If 6 miles Then 2 miles

✓ yes ✓ yes × no

(Jitendra, 2016)
Percent Change

When a new highway is built, the average time it takes for a bus to travel from one town to another is reduced from 25 minutes to 20 minutes. What is the percent decrease in time taken to travel between the two towns?

\[
\frac{\text{Original} - \text{New}}{\text{Original}} \times \frac{100}{1} = \frac{25 - 20}{25} \times \frac{100}{1} = \frac{5}{25} \times 100 = 20\%
\]

(Jitendra, 2016)