Schema Based Instruction

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Please work on the following…

If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

Variations of Set Up

$$\frac{300\text{ cal}}{x\text{ cal}} = \frac{100\text{ g}}{30\text{ g}}$$

$$\frac{300\text{ cal}}{100\text{ g}} = \frac{x\text{ cal}}{30\text{ g}}$$

$$\frac{x\text{ cal}}{300\text{ cal}} = \frac{30\text{ g}}{100\text{ g}}$$

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Original Problem:

If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

Variations:

a) How many calories are in 30 g of a certain food, given that there are 300 calories in 100 g of the same food?
b) A serving size of 100 g of a certain food has 300 calories. How many calories would a smaller serving size of 30 g have?
c) Your dad gives you a 30 ounce Hershey's dark chocolate mega-kiss in your lunch box. If a 100 ounce brick of Hershey's chocolate contains 300 micropops of caffeine, how much caffeine is in the mega-kiss?

What's the Deal?

What makes it difficult?

Students successful with word problems…

What’s the Key

- Understand the Problem
- Construct an accurate representation
- Generate, Plan, and Monitor the solution
- Execute the computation
- Meaningfully interpret the solution

What's the Key?

Students successful with word problems…

> distinguish "relevant information from irrelevant, perceiving rapidly and accurately the mathematical structure of the problems and generalizing across a wider range of mathematically similar problems."

(Van Dooren, 2010)

What’s the Key?

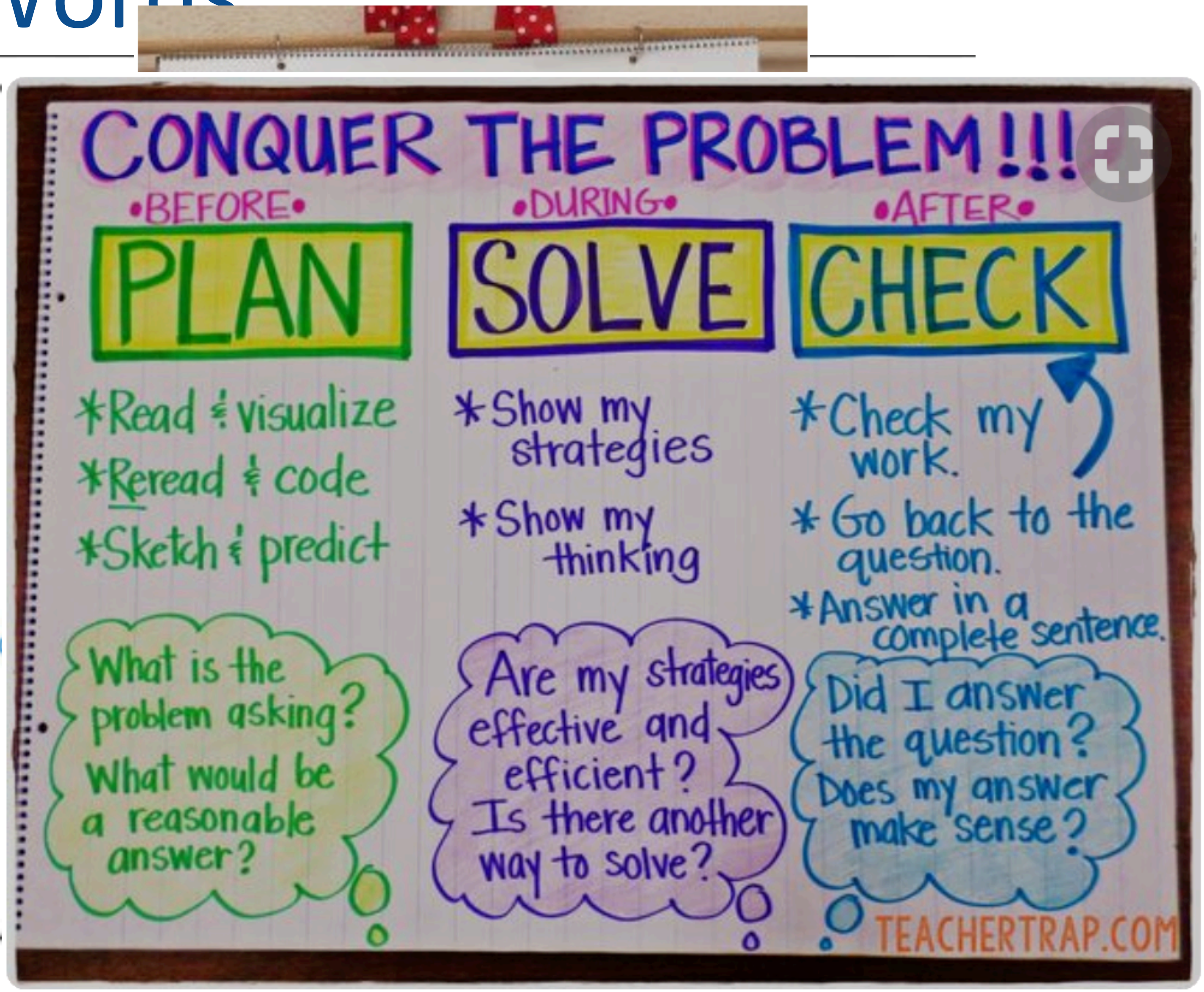
Students successful with word problems…

- Relevant vs. Irrelevant
- Structure across mathematically similar problems

(Van Dooren, 2010)

Key Words


C
U
B
E
S


CONQUER THE PROBLEM!!!
•BEFORE•
PLAN
*Read & visualize
*Reread & code
*Sketch & predict
What is the problem asking? What would be a reasonable answer?
•DURING•
SOLVE
*Show my strategies
*Show my thinking
Are my strategies effective and efficient? Is there another way to solve?
•AFTER•
CHECK
*Check my work.
*Go back to the question.
*Answer in a complete sentence.
Did I answer the question? Does my answer make sense?
TEACHERTRAP.COM

Traditional Approaches

- Key Words
- Draw a picture
- Heuristic
- Domain Specific Practice

Traditional Approaches

- Key Word Approach
  - Does not develop sense making
  - Lacks the structure to expand to more complicated problems
  - Words appear too often
  - Multi-Step problems (which begin in 2nd grade)

(Karp, 2017)

Draw a Picture

There are 4 adults and 2 children who need to cross the river. A small boat is available that can hold either 1 adult or 1 or 2 small children. Everyone can row the boat. How many one-way trips does it take for all of them to cross the river?

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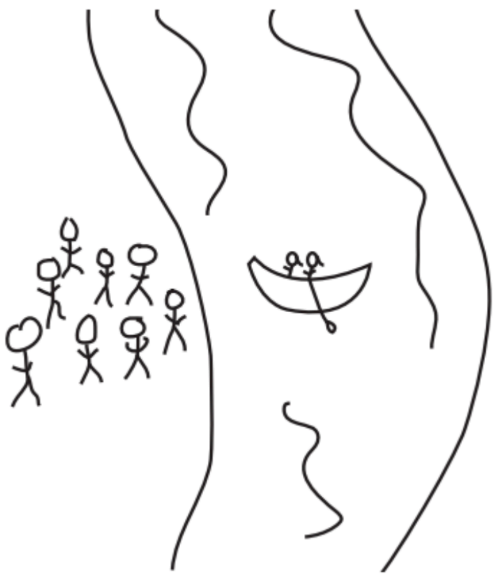

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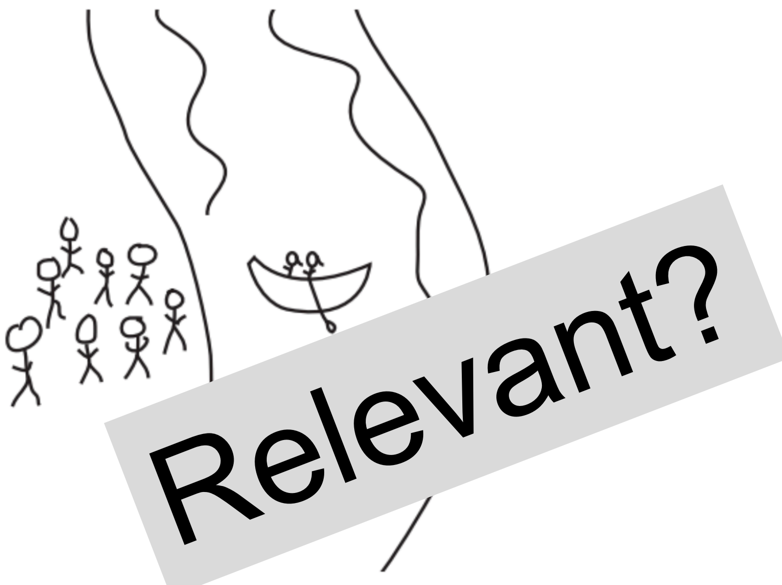


Schema’s Visual Representation

- Organize and summarize the info
- Make connections to the concrete
- Reasoning the story’s situations

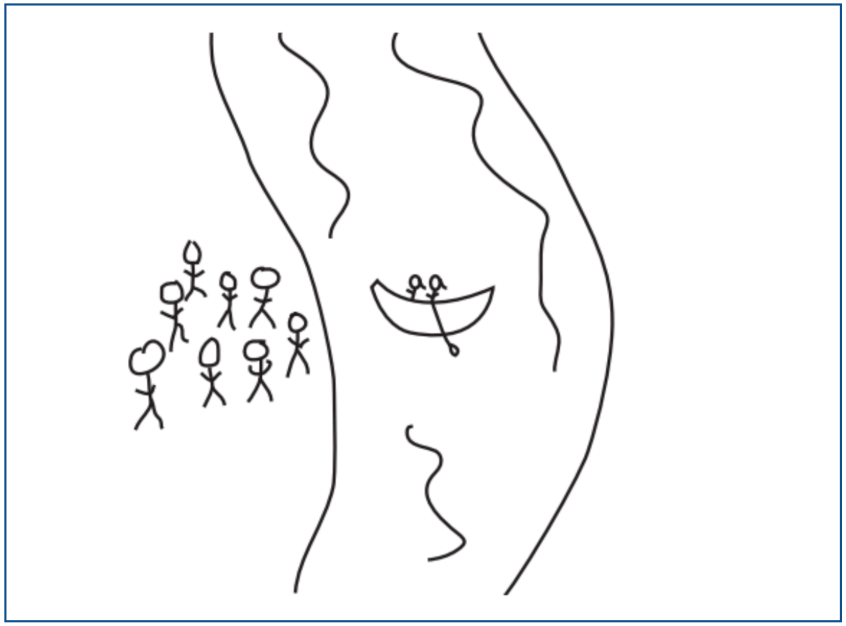
- All create reduction in working memory
- When attached to a schema, transfer becomes strong

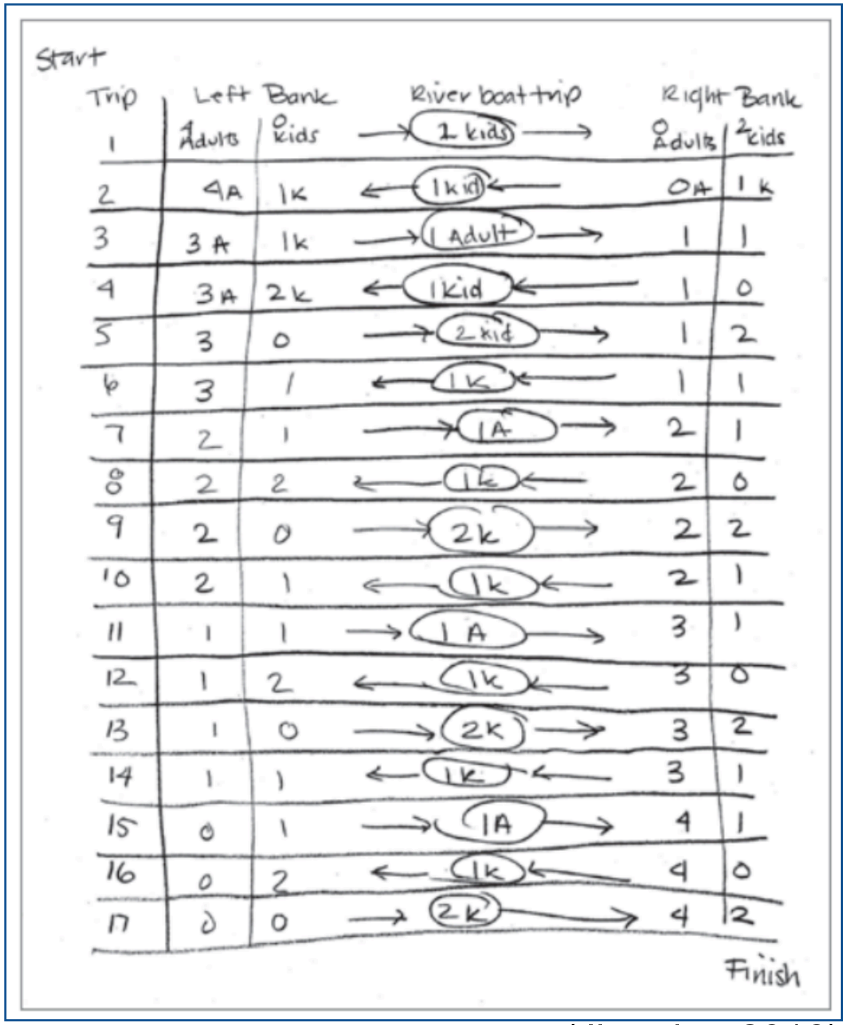
(Jitendra, 2013)

Schema vs Draw a Picture

- What is the difference?

Schema vs Draw a Picture





Start

Trip	Left Bank Adults	Left Bank Kids	River boat trip	Right Bank Adults	Right Bank Kids
	4	0		0	2
1			→ 2 kids →		
2	4A	1K	← 1 kid ←	0A	1 K
3	3 A	1k	→ 1 Adult →	1	1
4	3A	2k	← 1 Kid ←	1	0
5	3	0	→ 2 kid →	1	2
6	3	1	← 1K ←	1	1
7	2	1	→ 1A →	2	1
8	2	2	← 1K ←	2	0
9	2	0	→ 2k →	2	2
10	2	1	← 1k ←	2	1
11	1	1	→ 1 A →	3	1
12	1	2	← 1k ←	3	0
13	1	0	→ 2K →	3	2
14	1	1	← 1K ←	3	1
15	0	1	→ 1A →	4	1
16	0	2	← 1k ←	4	0
17	0	0	→ 2K →	4	2

Finish

(Jitendra, 2016)

Schematic Diagrams

- Highlight the relationships within the problem
- Establish deep understanding of the relationships
- Problems can be categorized by the student who understands these relationships
  - …"generalizing across a wider range of mathematically similar problems."

SBI (Schematic Based Instruction)

- Problem Schema Identification
- Representation
- Planning
- Solution

Conceptual Knowledge

- Problem Schema Identification
  - Schema Knowledge
- Representation
  - Elaboration Knowledge
- Planning
  - Strategic Knowledge
- Solution
  - Execution Knowledge

(Jitendra, 2013)

(Marshall, 1995)

Problem Schema Identification

- Organized structure of given elements and relations specific to a situation
- Bring clarity to the links of both the relationships and patterns of given operations

(Jitendra, 2013)

Problem Schema Identification

- What type of structure exists in the problem?

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and

(KSDE, 2018)

	Result Unknown	Change Unknown	Start Unknown
Change	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
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(KSDE, 2018)

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, $18 \div 3 = ?$	$? \times 6 = 18$, $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as

(KSDE, 2018)

Problem Schema Identification

- What type of structure exists in the problem?
  - Ratios
  - Proportions
  - Percents

Representation

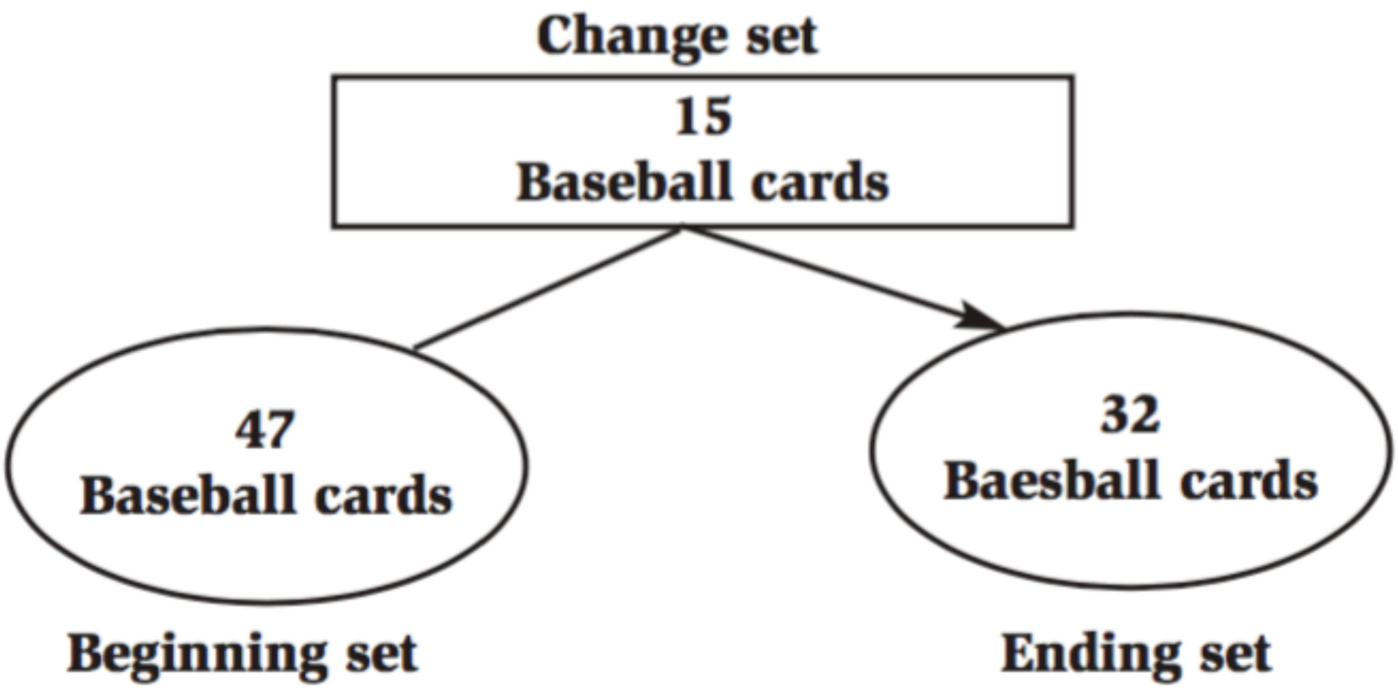
- Explicitly Taught ("Story Situation")
- Avoids irrelevant information
- Formats the problem
- Connects to the computation
- Vertically aligned

(Jitendra, 2013)

Story Situation

Change Story Situation

John had 47 baseball cards in his collection. He lost 15 of them when his family moved from Florida to New York. Now John has 32 baseball cards.

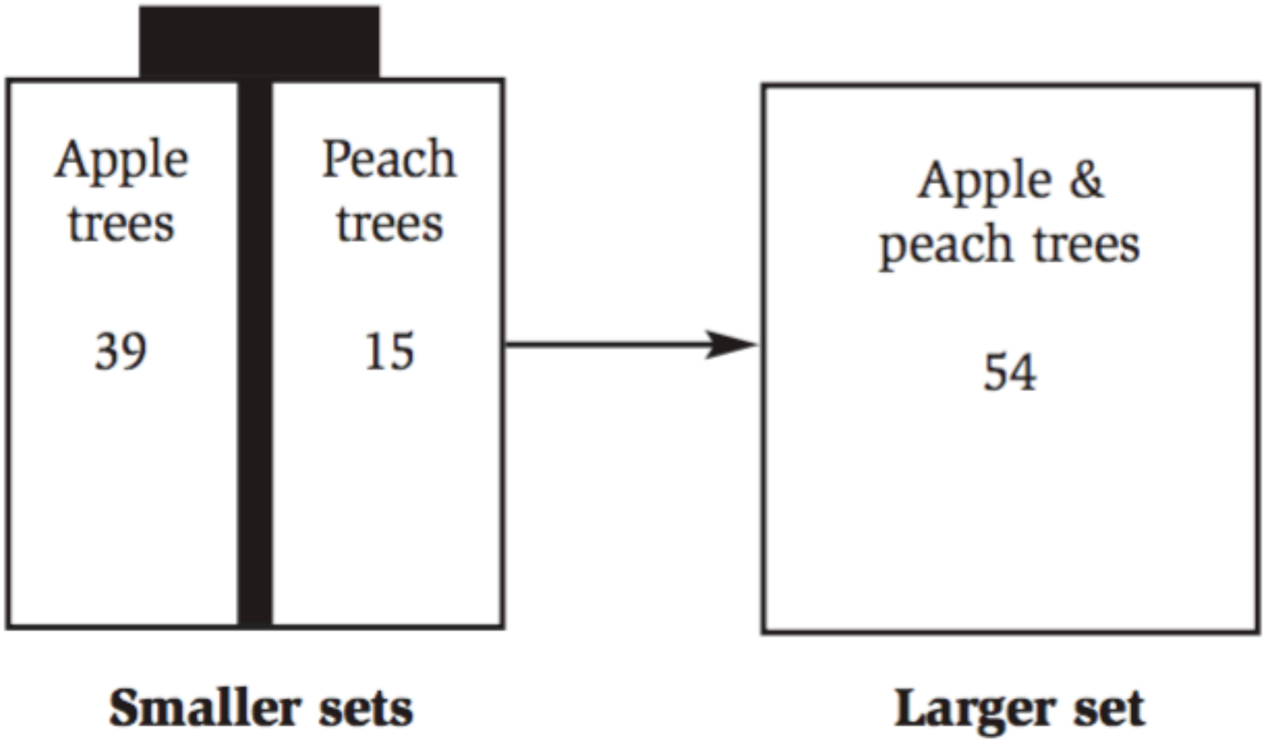


(Jitendra, 2002)

Story Situation

Group Story Situation

Tim has 54 fruit trees in his orchard. 39 are apple trees, and the remaining 15 are peach trees.

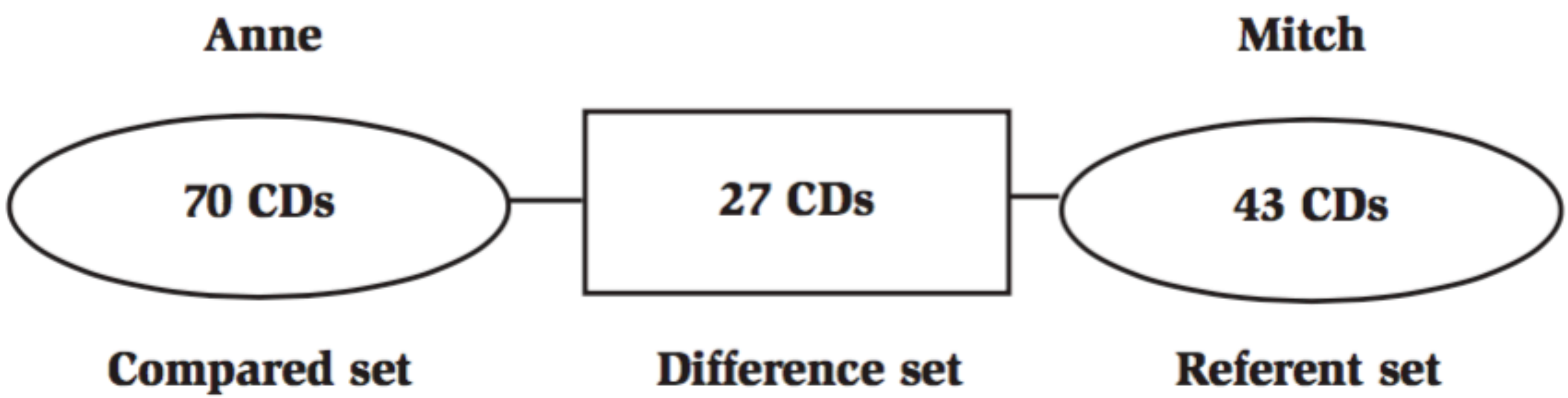


(Jitendra, 2002)

Story Situation

Compare Story Situation

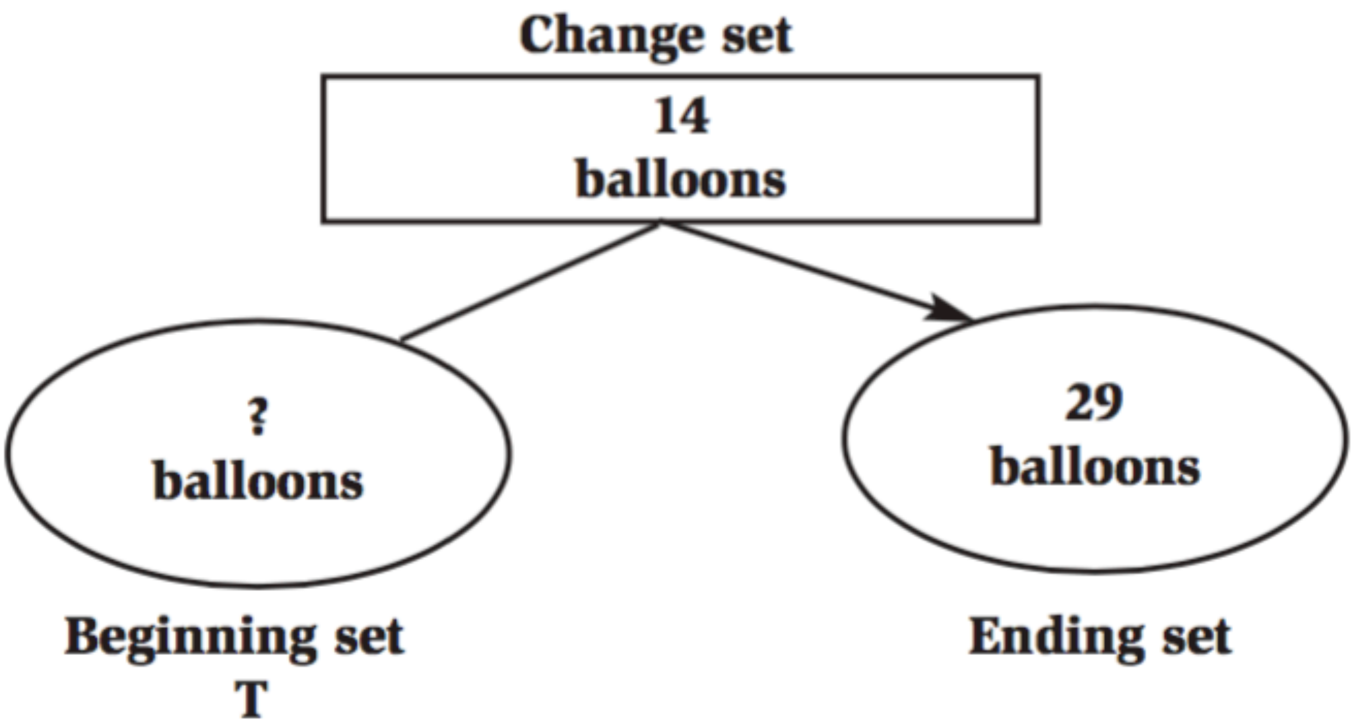
Mitch has 43 CDs and Anne has 70. Anne has 27 more CDs than Mitch.



(Jitendra, 2002)

Change Problem

A balloon man had some balloons. Then 14 balloons blew away and the man now has 29 balloons. How many balloons did the man begin with?



Total is not known, so add.

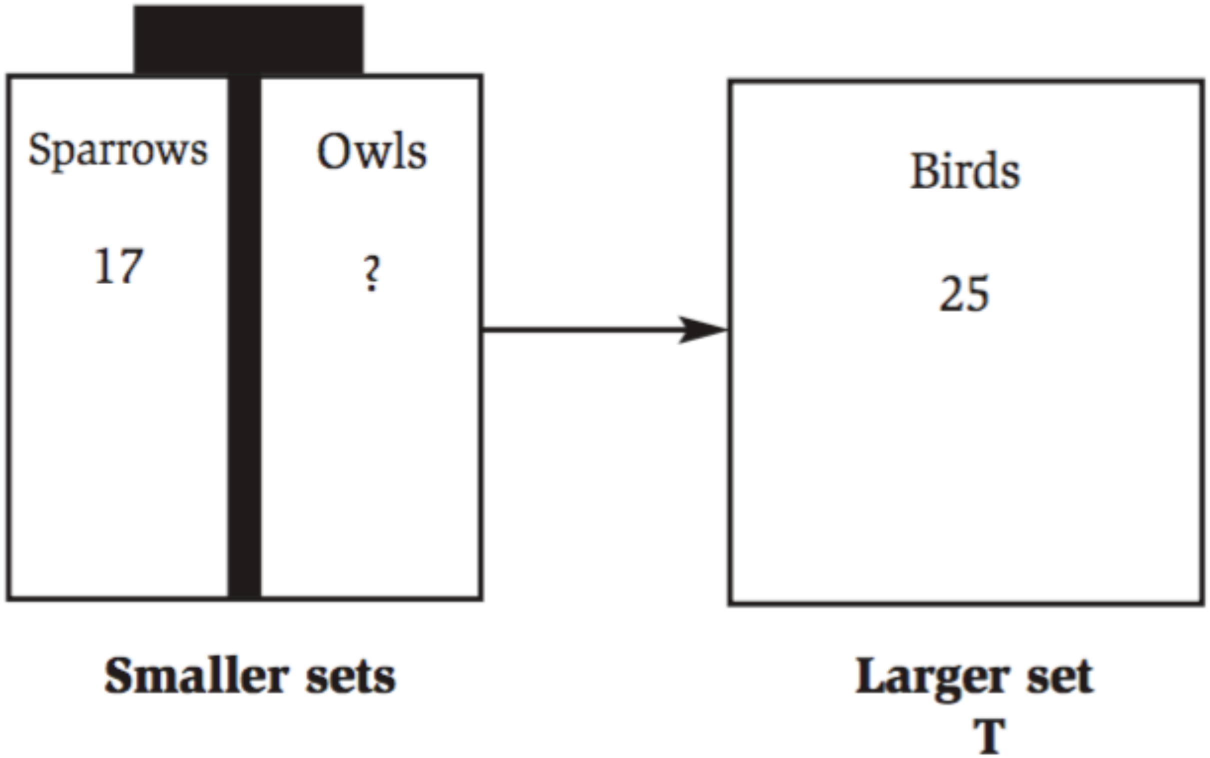
29 + 14 = 43

The man began with 43 balloons.

(Jitendra, 2002)

Group Problem

Jenny saw 25 birds on a camping trip. She saw 17 sparrows and some owls. How many owls did Jenny see on the camping trip?

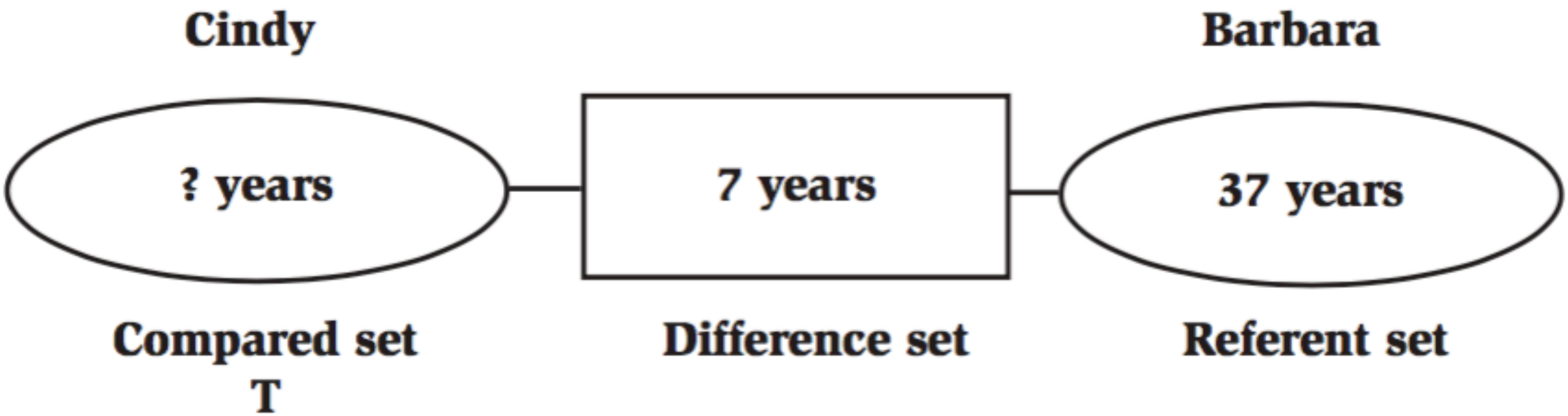


Total is known, so subtract.
25 - 17 = 8
Jenny saw 8 owls on the camping trip.

(Jitendra, 2002)

Compare Problem

Barbara is 37 years old. Cindy is 7 years older than Barbara. How old is Cindy?



Total is not known, so add.
$37 + 7 = 44$
Cindy is 44 years old.

(Jitendra, 2002)

Planning

- Self-Monitor from start
  - Think about the needed solution
    - Estimate the solution (not too exact)
    - Rewrite the question
  - Determine Structure and Model
  - Plan the computation strategy (ie – Subtract the lesser, cross-multiply, equivalent fractions, etc.)
  - Solve the Computation

(Jitendra, 2013)

Solution

- What does the computation mean?
- How close is my estimate?
- Does my answer make sense?
- Does it answer the question being asked?

Try to find Varying Diagrams to Explore and Discuss

- Examples and links from Free resources

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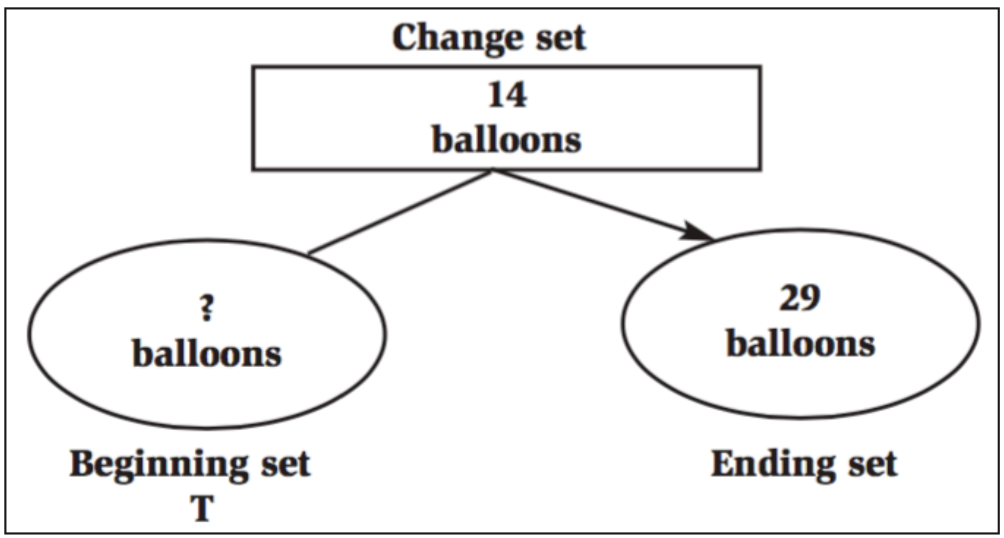

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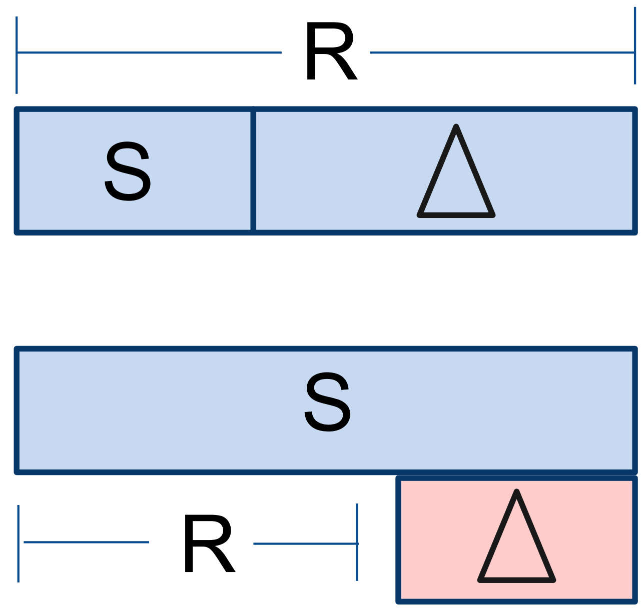

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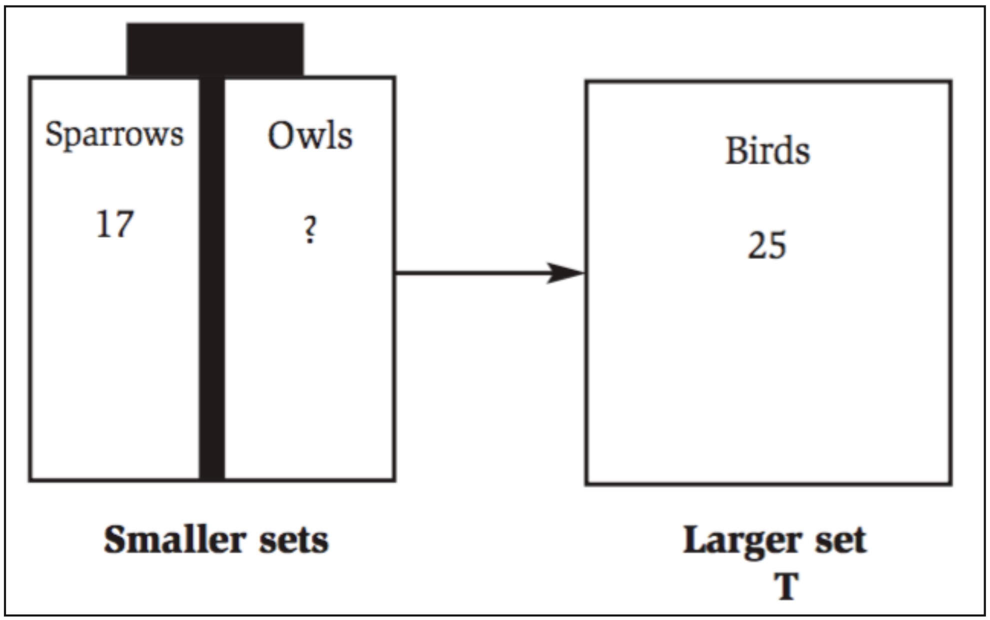

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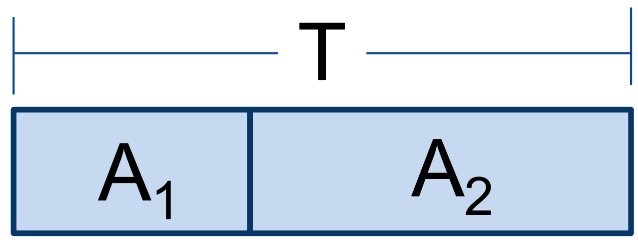

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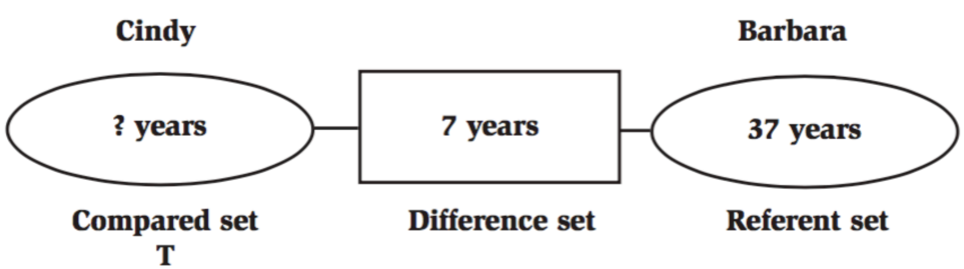

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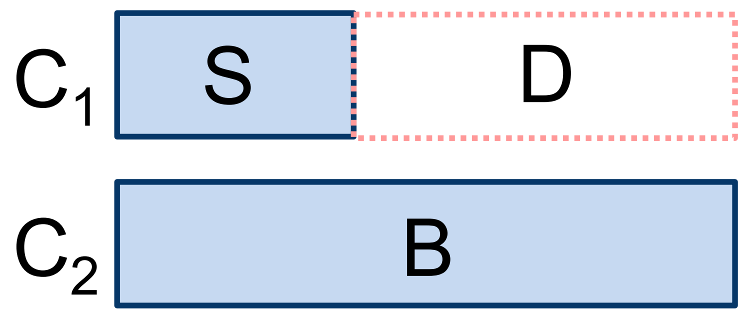
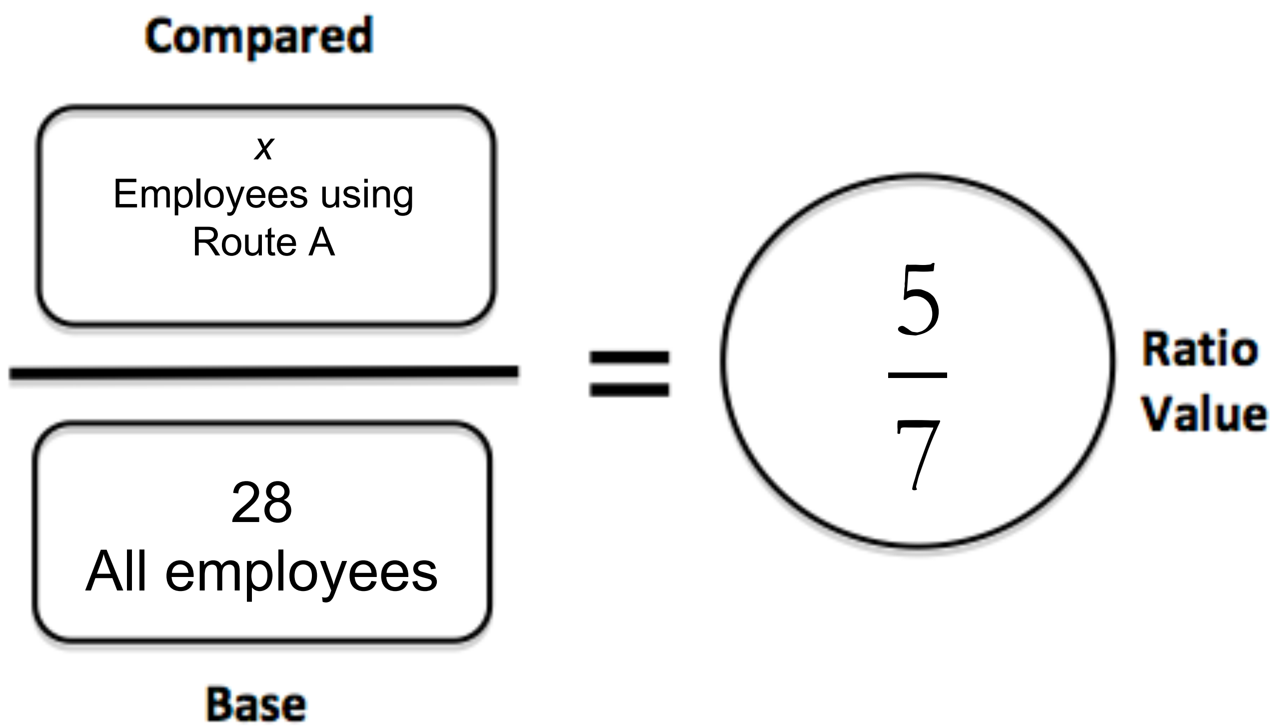


Ratio Problems

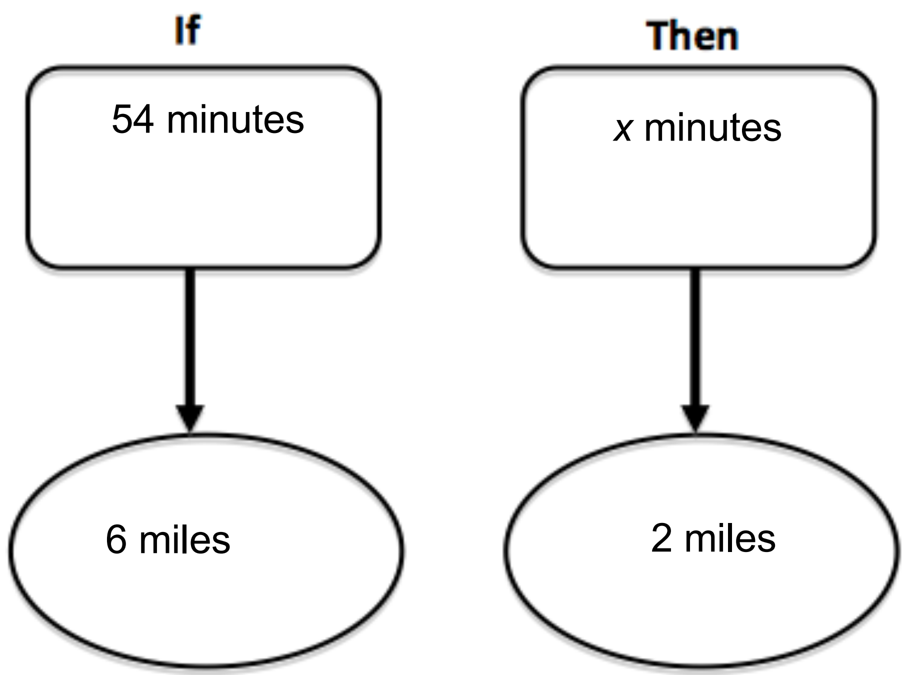
Problem 2: There are 28 employees at the local bank. Every morning 5 out of 7 employees use Route A to drive to work. How many employees use Route A to drive to work? (Problem 4.3)



(Jitendra, 2016)

Proportion Problem

Carlos is on the school's track team. He takes 54 minutes to run 6 miles. Assuming that he runs at a constant pace for all 6 miles, how long did it take him to run 2 miles? (Problem 7.2)



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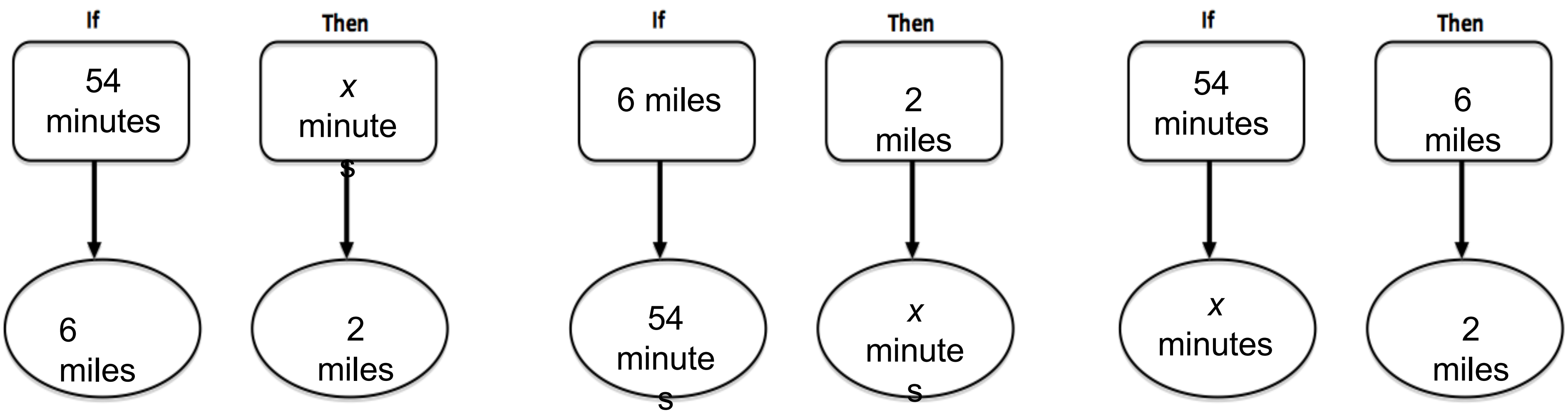

✓ yes ✓ yes ✗ no

(Jitendra, 2016)

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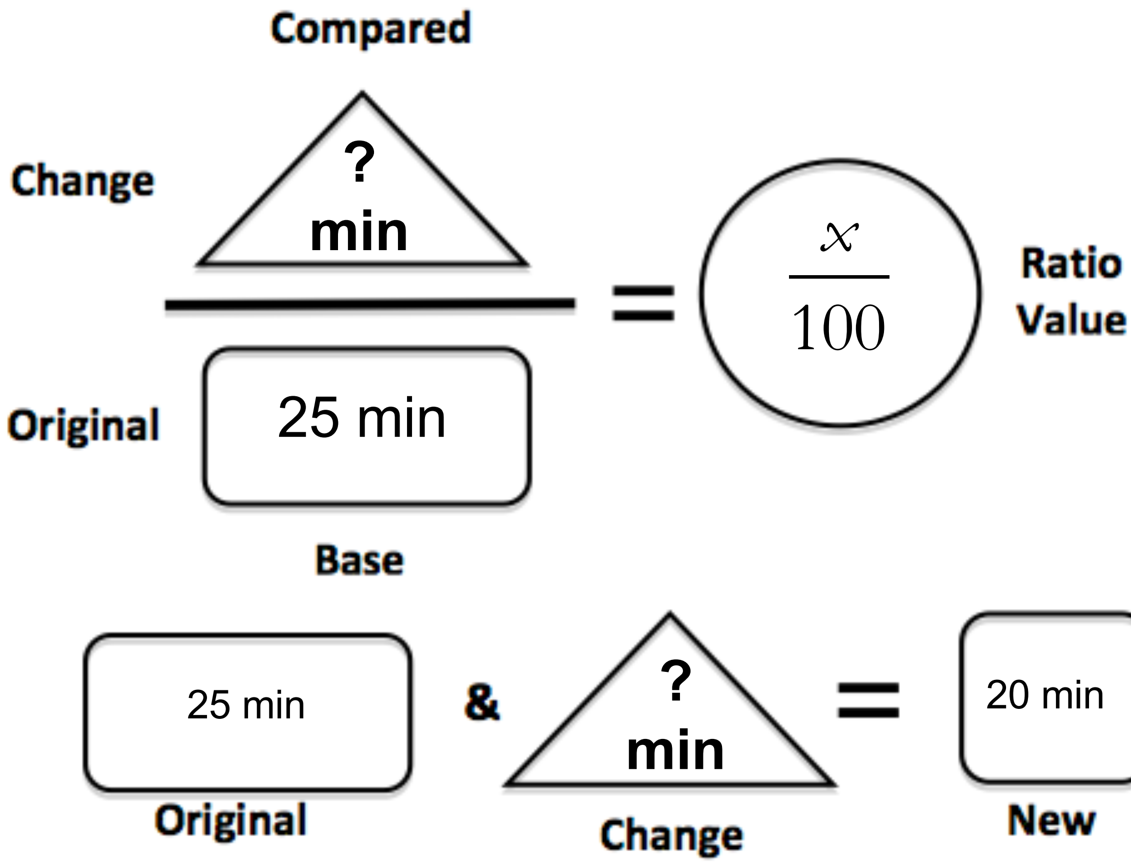
Percent Change

When a new highway is built, the average time it takes for a bus to travel from one town to another is reduced from 25 minutes to 20 minutes. What is the percent decrease in time taken to travel between the two towns?



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Resources